Fast Flow Formulations for Sparse QAP Instances

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.

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Quadratic Assignment Problems

Quadratic Assignment Problems

- Problem definition.
- Bounds and Linearizations.
- Fast Flow Formulations.
- Computational Experience.
- A Facility Lay-Out Problem.
- Conclusions

Problem Definition

Motivation

- Optimized resource allocation after World War II was a great challenge.
- Cold War aspect: Captalism should be theoretically feasible.
- Koopmans and Beckmann: First work to devise and define the problem.

The Simplest Locational Model: Linear Assignment

(1) $$\max p = \sum_{k=1}^{n} \sum_{i=1}^{n} a_{ki} x_{ki}$$

subject to:

(2) $$\sum_{k=1}^{n} x_{ki} = 1, \quad \forall\ i = 1, \ldots, n$$

(3) $$\sum_{i=1}^{n} x_{ki} = 1, \quad \forall\ k = 1, \ldots, n$$

(4) $$x_{ki} \in \{0, 1\}, \quad \forall\ k, i = 1, \ldots, n.$$

Linear Assignment Features

- Easy to read: His dual is the same as the Transportation Problem.
- Easy to Solve: The Hungarian Method is a great tool for LAP.
- Drawback: Is too simple for real-world applications.

The Great Idea

QAP = LAP + Interdependency.

Measuring Interdependency

$$q = \sum_{(k,l)} \sum_{(i,j)} b_{kl} x_{ki} c_{ij} x_{lj} \tag{5}$$

Interdependency is evaluated by flows of intermediate commodities among different economic activities.

Quadratic Assignment Features

- Hard to read: It is not simple how to interpret his dual information.
- Hard to Solve: Strong formulations too hard.
- Hard to Solve: Easy formulations too weak.
- Hard to Solve: If $n \geq 20$ the instance is considered very hard.

Bounds and Linearizations

Bounds and Linearizations

- Koopmans and Beckmann Flow Formulation.
- Gilmore and Lawler Bound.
- Lawler formulation.
- Frieze and Yadegar Formulation.
- Adams and Johnson Formulation.

Koopmans and Beckmann Flow Formulation

$$(6) \qquad \max \sum_{k=1}^{n} \sum_{i=1}^{n} a_{ki} x_{ki} - \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} c_{ij} f_{ij}^{kl}$$

subject to (2) - (4) e:

$$(7) \quad b_{kl} x_{ki} + \sum_{j=1}^{n} f_{ji}^{kl} = b_{kl} x_{li} + \sum_{j=1}^{n} f_{ij}^{kl} , \ \forall \ i,k,l = 1,...,n$$

$$(8) \quad f_{ii}^{kl} = 0 , \ \forall \ i,k,l = 1,...,n$$

$$(9) \quad f_{ij}^{kl} \geq 0 , \ \forall \ i,j,k,l = 1,...,n$$

Drawback: Is too weak for the problem

Gilmore and Lawler Bound

$$\max p = \sum_{k=1}^{n} \sum_{i=1}^{n} (a_{ki} - \lambda_{ki}) x_{ki} \tag{10}$$

subject to:

$$\sum_{k=1}^{n} x_{ki} = 1, \quad \forall \; i = 1, ..., n \tag{11}$$

$$\sum_{i=1}^{n} x_{ki} = 1, \quad \forall \; k = 1, ..., n \tag{12}$$

$$x_{ki} \in \{0, 1\}, \quad \forall \; k, i = 1, ..., n. \tag{13}$$

Gilmore-Lawler Bound

$$\lambda_{ki} = \max \sum_{l=1}^{n} \sum_{j=1}^{n} b_{kl} c_{ij} x_{lj} \tag{14}$$

subject to:

$$\sum_{j=1}^{n} x_{lj} = 1\ ,\ \forall l = 1, ..., n, \tag{15}$$

$$\sum_{l=1}^{n} x_{lj} = 1\ ,\ \forall j = 1, ..., n, \tag{16}$$

$$x_{ki} = 1\ , \tag{17}$$

$$x_{lj} \geq 0\ ,\ \forall l, j = 1, ..., n. \tag{18}$$

GLB takes $n^2 + 1$ LAP solutions. It is tight for $n \leq 20$.

Lawler Formulation

$$(19) \quad \max \sum_{k=1}^{n}\sum_{i=1}^{n} a_{ki}x_{ki} - \sum_{k=1}^{n}\sum_{l=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n} d_{kilj}y_{kilj}$$

subject to (2) - (4) e:

$$(20) \quad \sum_{k,l,i,j=1}^{n} y_{kilj} = n^2$$

$$(21) \quad y_{kilj} \leq x_{ki}\ ,\ \forall i,j,k,l = 1,\dots,n,\ i \neq j,\ k \neq l$$

$$(22) \quad y_{kilj} \leq x_{lj}\ ,\ \forall i,j,k,l = 1,\dots,n,\ i \neq j,\ k \neq l$$

$$(23) \quad y_{kilj} \geq 0\ ,\ \forall i,j,k,l = 1,\dots,n,\ i \neq j,\ k \neq l$$

The central idea: $x_{ki}x_{lj} = y_{kilj}$. Here $d_{ijkl} = b_{kl}c_{ij}$.
Too large: n^4 variables and n^4 linking constraints.

Frieze and Yadegar Formulation

$$\text{(24)} \qquad \max \sum_{k=1}^{n}\sum_{i=1}^{n} a_{ki}x_{ki} - \sum_{k=1}^{n}\sum_{l=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n} d_{kilj}y_{kilj}$$

subject to (2) - (4) e:

$$\text{(25)} \qquad \sum_{j=1}^{n} y_{kilj} = x_{ki}\ ,\ \forall i,k,l = 1,\dots,n,\ i \neq j,\ k \neq l$$

$$\text{(26)} \qquad \sum_{l=1}^{n} y_{kilj} = x_{ki}\ ,\ \forall i,j,k = 1,\dots,n,\ i \neq j,\ k \neq l$$

$$\text{(27)} \qquad \sum_{k=1}^{n} y_{kilj} = x_{lj}\ ,\ \forall i,j,l = 1,\dots,n,\ i \neq j,\ k \neq l$$

$$\text{(28)} \qquad \sum_{i=1}^{n} y_{kilj} = x_{lj}\ ,\ \forall j,l,k = 1,\dots,n,\ i \neq j,\ k \neq l$$

$$\text{(29)} \qquad y_{kilj} \geq 0\ ,\ \forall i,j,k,l = 1,\dots,n,\ i \neq j,\ k \neq l$$

Another big model: n^4 variables and $4n^3$ linking constraints.

Adams and Johnson Formulation

$$(30) \quad \max \sum_{k=1}^{n}\sum_{i=1}^{n} a_{ki}x_{ki} - \sum_{k=1}^{n}\sum_{l=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n} d_{kilj}y_{kilj}$$

subject to (2) - (4) e:

$$(31) \quad \sum_{j=1}^{n} y_{kilj} = x_{ki}\ ,\ \forall i,k,l = 1,...,n,\ i \neq j,\ k \neq l$$

$$(32) \quad \sum_{l=1}^{n} y_{kilj} = x_{ki}\ ,\ \forall i,j,k = 1,...,n,\ i \neq j,\ k \neq l$$

$$(33) \quad y_{kilj} = y_{ljki}\ ,\ \forall i,j,k,l = 1,...,n,\ i \neq j,\ k \neq l,$$

$$(34) \quad y_{kilj} \geq 0\ ,\ \forall i,j,k,l = 1,...,n,\ i \neq j,\ k \neq l$$

Yet another large formulation: n^4 variables and $n^4 + 2n^3$ linking constraints.
As tight as Frieze and Yadegar formulation until we are aware.

Important Remarks

- The main difficulty with QAP appears to be the excessive symmetry.
- A good alternative to avoid this symmetry is to include linear coefficients.
- It is necessary to find a formulation that balances bound quality and computational effort.

What if we could find a formulation that explores the non null linear term in the best way?
What if this formulation combines a bound not so poor being also easy to solve?

Fast Flow Formulations

Fast Flow Formulations

- Miranda and Luna Enhanced Flow Formulation.
- Preprocessing for CPLEX.
- Improving Bounds.
- Re-interpreting Adams and Johnson.

Miranda and Luna: Enhanced Flow Formulation

$$(35) \qquad \max \sum_{k=1}^{n} \sum_{i=1}^{n} a_{ki} x_{ki} - \sum_{(i,j),i \neq j} \sum_{(k,l),k \neq l} c_{ij} f_{ij}^{kl}$$

subject to (2) - (4) and:

$$(36) \quad -\sum_{j=1}^{n} f_{ij}^{kl} = -b_{kl} x_{ki} \ , \ \forall \ i,k,l = 1,...,n, i \neq j \ , k \neq l$$

$$(37) \quad \sum_{i=1}^{n} f_{ij}^{kl} = b_{kl} x_{lj} \ , \ \forall \ j,k,l = 1,...,n, i \neq j \ , k \neq l$$

$$(38) \quad b_{lk} \ f_{ij}^{kl} = b_{kl} f_{ji}^{lk} \ , \ \forall \ i,j,k,l = 1,...,n, i \neq j \ , k \neq l$$

$$(39) \quad f_{ij}^{kl} \geq 0 \ , \ \forall \ i,j,k,l = 1,...,n, i \neq j \ , k \neq l$$

Preprocessing for CPLEX

$$(40)\max \sum_{k=1}^{n}\sum_{i=1}^{n} a_{ki}x_{ki} - \sum_{i\neq j}\sum_{(k,l)\in\phi}[c_{ij}b_{kl} + c_{ji}b_{lk}]f_{ij}^{kl}$$

subject to (2) - (4) and:

$$(41)\quad \sum_{j\neq i} f_{ij}^{kl} = x_{ki}\ ,\ \forall\ i = 1,...,n,\ \ \forall(k,l)\in\ \phi$$

$$(42)\quad \sum_{i\neq j} f_{ij}^{kl} = x_{lj}\ ,\ \forall\ j = 1,...,n,\ \ \forall(k,l)\in\ \phi$$

$$(43)\quad f_{ij}^{kl} \geq 0\ ,\ \forall\ i,j = 1,...,n, i\neq j\ ,\ \forall(k,l)\in\phi$$

where:

$$(44)\quad \phi = \{(k,l)|k<l\ \wedge\ (b_{kl}\neq 0\ \vee b_{lk}\neq 0)\}$$

Improving Bounds

If the bound is too poor in a given instance, we can add two families of valid inequalities. These inequalities are transliterations of Frieze and Yadegar work.

$$(45) \sum_{k,\ (k,l)\in\phi} f_{ij}^{kl} \leq x_{lj}\ ,\ \forall\ i,j = 1,...,n,\ i \neq j\ ,\ \forall l,\ (k,l) \in\ \phi$$

$$\sum_{l,\ (k,l)\in\phi} f_{ij}^{kl} \leq x_{ki}\ ,\ \forall\ i,j = 1,...,n,\ i \neq j\ ,\ \forall k,\ (k,l) \in\ \phi$$

Re-interpreting Adams and Johnson

It is possible to re-interpret Adams and Johnson formulation as a flow formulation. The idea is to take advantage of sparsity without compromising LP bounds.

$$(46) \quad \max \sum_{k=1}^{n}\sum_{i=1}^{n} a_{ki}x_{ki} - \sum_{i\neq j}\sum_{(k,l)\in\phi} [c_{ij}b_{kl} + c_{ji}b_{lk}]f_{ij}^{kl}$$

subject to (2) - (4) and:

$$(47) \quad \sum_{j\neq i} f_{ij}^{kl} = x_{ki} \,,\ \forall\, i = 1,...,n,\ \ \forall (k,l) \in \phi$$

$$(48) \quad \sum_{i\neq j} f_{ij}^{kl} = x_{lj} \,,\ \forall\, j = 1,...,n,\ \ \forall (k,l) \in \phi$$

$$(49) \quad \sum_{l\in\phi | k<l} f_{ij}^{kl} + \sum_{l\in\phi | k>l} f_{ji}^{lk} \leq x_{ki} \,,\ \forall\, k \in K,\ \forall i \neq j$$

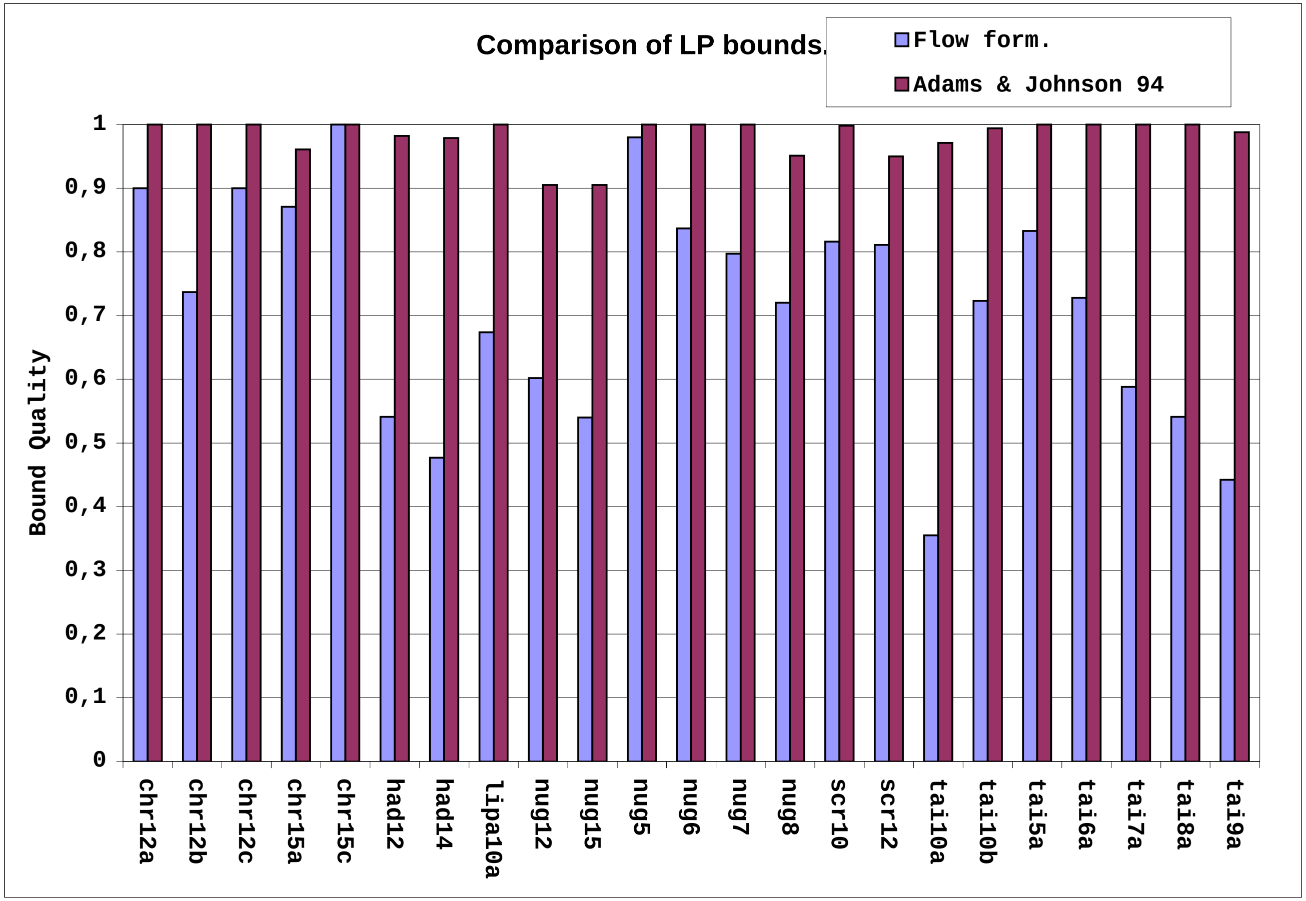
$$(50) \quad f_{ij}^{kl} \geq 0 \,,\ \forall\, i,j = 1,...,n, i \neq j \,,\ \forall (k,l) \in \phi$$

Computational Experience

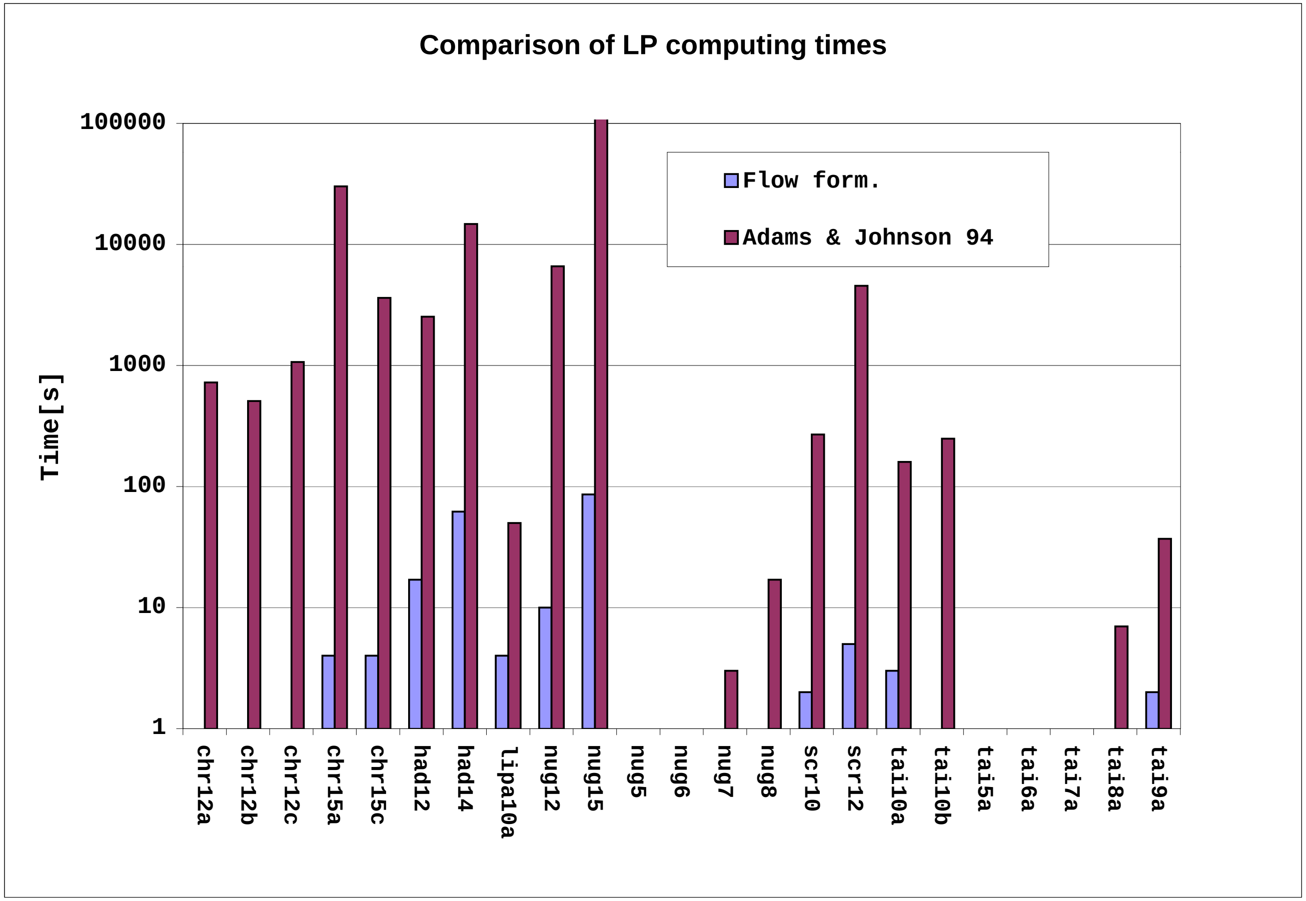
Comparisons

- LP Bounds: AJ94 x ML04.
- Computing Times: AJ94 x ML04.
- LP Bounds: GLB x ML04.
- Computing times: GLB x ML04.

LP Bounds: AJ94 x ML04



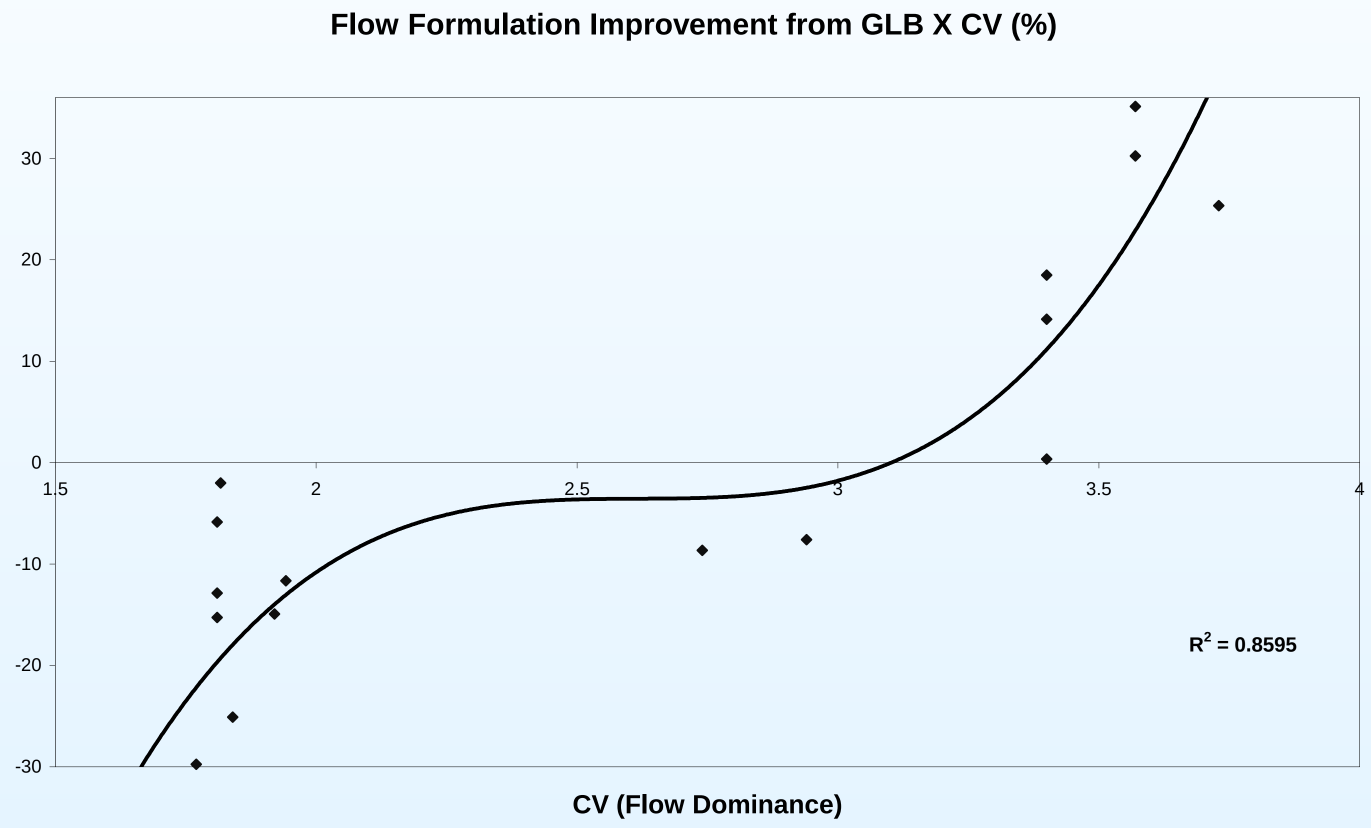
Computing Times (LP): AJ94 x ML04.



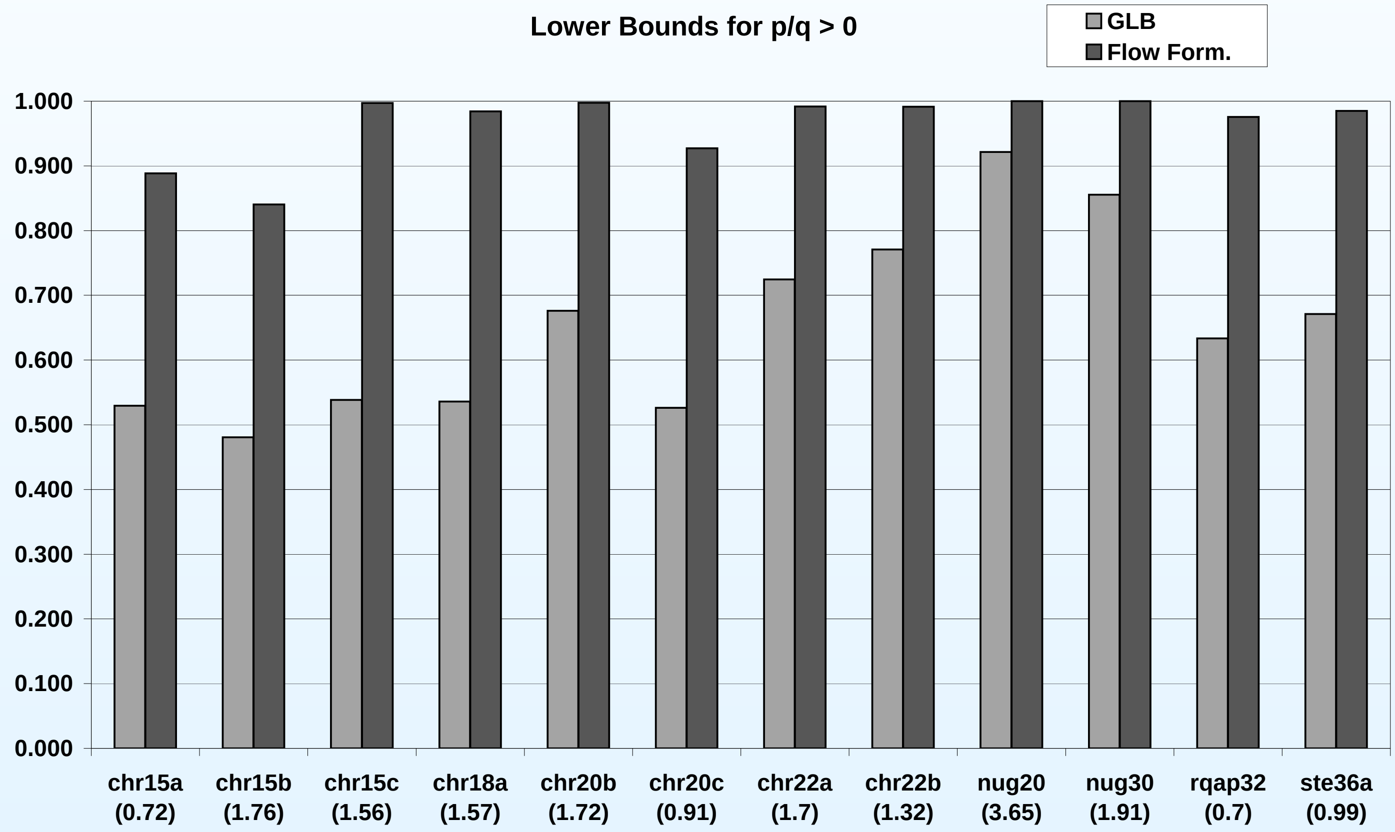
Computing Times (MIP): AJ94 x ML04.

Original instance	Problem size	Variables		p/q ratio	Flow form. time[s]	Adams and Johnson time[s]
		Integer	Continuous			
mchr12a				0.426	4	584
mchr12b	12	144	17424	0.383	5	516
mchr12c				0.314	3	803
mchr15a				0.631	16	14209
mchr15a	15	225	44100	0.856	6	10125
mchr15c				0.807	9	3058
mchr18a				0.564	116	*
mchr18b	18	324	93636	1.864	7	*
mchr20a				1.047	76	*
mchr20b	20	400	144400	0.824	47	*
mchr20c				1.056	108	*
mchr22a	22	484	213444	0.304	138	*
mchr22b				0.293	96	*

LP Bounds: GLB x ML04



LP Bounds: GLB x ML04



Computing Times (MIP): GLB x ML04.

Original Instance	Instance size	Variables		p/q ratio	Flow form. time[s]	Branch-and-Bound (GLB) time[s]
		Integer	Continuous			
mchr15b	15	225	44100	0.726	36	0
mchr18a	18	324	93636	1.304	41	26
mchr20c	20	400	144400	0.707	54	75
mchr22a	22	484	213444	0.744	19	11
mnug20	20	400	144400	3.268	42	0
mnug30	30	900	756900	1.358	1779	23
rqap32	32	1024	984064	0.701	1964	2503
rqap33	33	1089	1115136	0.97	3236	3060
rqap34	34	1156	1258884	0.74	7207	7198
rqap35	35	1225	1416100	0.87	2610	16154
mste36a				0.687	2457	27974
mste36a	36	1296	1587600	1.199	2801	2925
mtho40				0.741	10532	*
mtho40	40	1600	2433600	1.040	7689	1984

After Preprocessing

After Preprocessing:

Tempos de Computação (MIP): GLB x CML04

Original Instance	Instance size	p/q ratio	Flow Form. Time [s]	Branch-And-Bound (GLB) Time [s]	Compact Flow Form. Time [s]
mchr15a		0.631	16	0	14
mchr15b	15	0.726	36	0	9
mchr15c		0.807	9	0	8
mchr18a		0.564	116	83	**74**
mchr18a	18	1.304	41	26	**22**
mchr20b		1.702	10	15	**13**
mchr20c	20	0.707	54	75	**36**
mchr22a		0.744	19	11	**2**
mchr22b	22	1.341	18	4	6
mnug20	20	3.268	42	0	4
mnug20		4.477	80	0	4
mnug30	30	1.358	1779	23	200
rqap32	32	0.701	1964	2503	**1306**
rqap33	33	0.97	3236	3060	**1735**
rqap34	34	0.74	7207	7198	**3417**
rqap35	35	0.87	2610	16154	**1956**
mste36a	36	1.199	2801	2925	**325**
mtho40		0.741	10532	*	**5362**
mtho40	40	1.040	7689	1984	**1551**
msko42		0.854	*	*	**16676**
msko42	42	1.280	*	*	**3724**

After Improving Bounds

After Improving Bounds:

Limites de PL: GLB x SCML04

instance	ML04		Bound	SCML04		Bound	AJ94		Bound
	lp bound	time[s]	Quality	lp bound	time[s]	Quality	lp bound	time[s]	Quality
chr12a.dat	8593.12	1	0.900	9246.67	1	0.968	9552	725	1.000
chr12b.dat	7184	1	0.737	9742	1	1.000	9742	508	1.000
chr12c.dat	10042.7	1	0.900	10121.1	1	0.907	11156	1068	1.000
chr15a.dat	8621.94	4	0.871	9295.33	4	0.939	9513	30146	0.961
chr15c.dat	9504	4	1.000	9504	4	1.000	9504	3622	1.000
had12.dat	894	17	0.541	1543.78	1414	0.934	1621.54	2533	0.982
had14.dat	1300.5	62	0.477	2583.07	9269	0.948	2666.12	14778	0.979
lipa10a.dat	318.8	4	0.674	448.123	161	0.947	473	50	1.000
nug12.dat	348	10	0.602	471.821	371	0.816	522.89	6597	0.905
nug15.dat	621	86	0.540	912.063	8790	0.793	1041	131923	0.905
nug5.dat	49	1	0.980	50	1	1.000	50	1	1.000
nug6.dat	72	1	0.837	80	1	0.930	86	1	1.000
nug7.dat	118	1	0.797	134.438	1	0.908	148	3	1.000
nug8.dat	154	1	0.720	191	3	0.893	203.5	17	0.951
scr10.dat	21958	2	0.816	24245.8	9	0.901	26873.1	269	0.998
scr12.dat	25474	5	0.811	27368.7	42	0.871	29827.3	4555	0.950
tai10a.dat	47953.3	3	0.355	112221	70	0.831	131098	160	0.971
tai10b.dat	855788	1	0.723	1139856.99	30	0.963	1176140	248	0.994
tai5a.dat	10747	1	0.833	12902	1	1.000	12902	1	1.000
tai6a.dat	21427.8	1	0.728	28013.4	1	0.952	29432	1	1.000
tai7a.dat	31730.1	1	0.588	47948	1	0.888	53976	1	1.000
tai8a.dat	41952.2	1	0.541	69705.1	1	0.899	77502	7	1.000
tai9a.dat	41816	2	0.442	83180.9	24	0.879	93501	37	0.988

Preprocessing Adams and Johnson

Preprocessing AJ94:

Limites de PL: [AJ94] Versus [CRLT-level1]

Instance		Compact RLT level1			RLT level 1 [AJ94]		
name	size	lp bound	time [s]	bound quality	lp bound	time [s]	bound quality
nug5	5	50.0	0	1.0000	50.0	1	1.0000
tai5a	5	12,902.0	0	1.0000	12,902.0	1	1.0000
nug6	6	86.0	0	1.0000	86.0	1	1.0000
tai6a	6	29,432.0	1	1.0000	29,432.0	1	1.0000
nug7	7	148.0	1	1.0000	148.0	3	1.0000
tai7a	7	53,976.0	1	1.0000	53,976.0	1	1.0000
nug8	8	202.6	6	0.9469	203.5	17	0.9509
tai8a	8	77,502.0	5	1.0000	77,502.0	7	1.0000
tai9a	9	93,501.0	26	0.9882	93,501.0	37	0.9882
lipa10a	10	473.0	50	1.0000	47 3.0	50	1.0000
scr10	10	26,659.0	12	0.9902	26,873.1	269	0.9982
tai10a	10	131,053.0	86	0.9706	131,098.0	160	0.9709
tai10b	10	1,172,043.4	47	0.9901	1,176,140.0	248	0.9936
chr12a	12	9,552.0	2	1.0000	9,552.0	725	1.0000
chr12b	12	9,742.0	1	1.0000	9,742.0	508	1.0000
chr12c	12	10,927.3	1	0.9795	11,156.0	1,068	1.0000
had12	12	1,621.5	1,633	0.9816	1,621.5	2,533	0.9816
nug12	12	522.7	308	0.9043	522.9	6,597	0.9047
scr12	12	29,766.5	59	0.9477	29,827.3	4,555	0.9496
had14	14	2,666.1	12,044	0.9788	2,666.1	14,778	0.9788
chr15a	15	9,456.5	3	0.9556	9,513.0	30,146	0.9613
chr15c	15	9,504.0	2	1.0000	9,504.0	3,622	1.0000
nug15	15	1,040.3	7,895	0.9046	1,041.0	131,923	0.9052
nug20	20	2,180.4	157,424	0.8484	2,181.6	2,310,985	0.8489

A Facility Lay-Out Problem

Problem Description

- SILA BRASIL is an automotive components manufacturer in Belo Horizonte, Minas Gerais, Brasil.
- SILA is a part of FIAT S.A. supply chain.
- Its expertise is the manufacturing of precision components for 16-valve engines and other machine elements.

Problem Description

- The main problem in its former lay-out was the excessive traffic of vehicles inside his plant to transport raw materials.

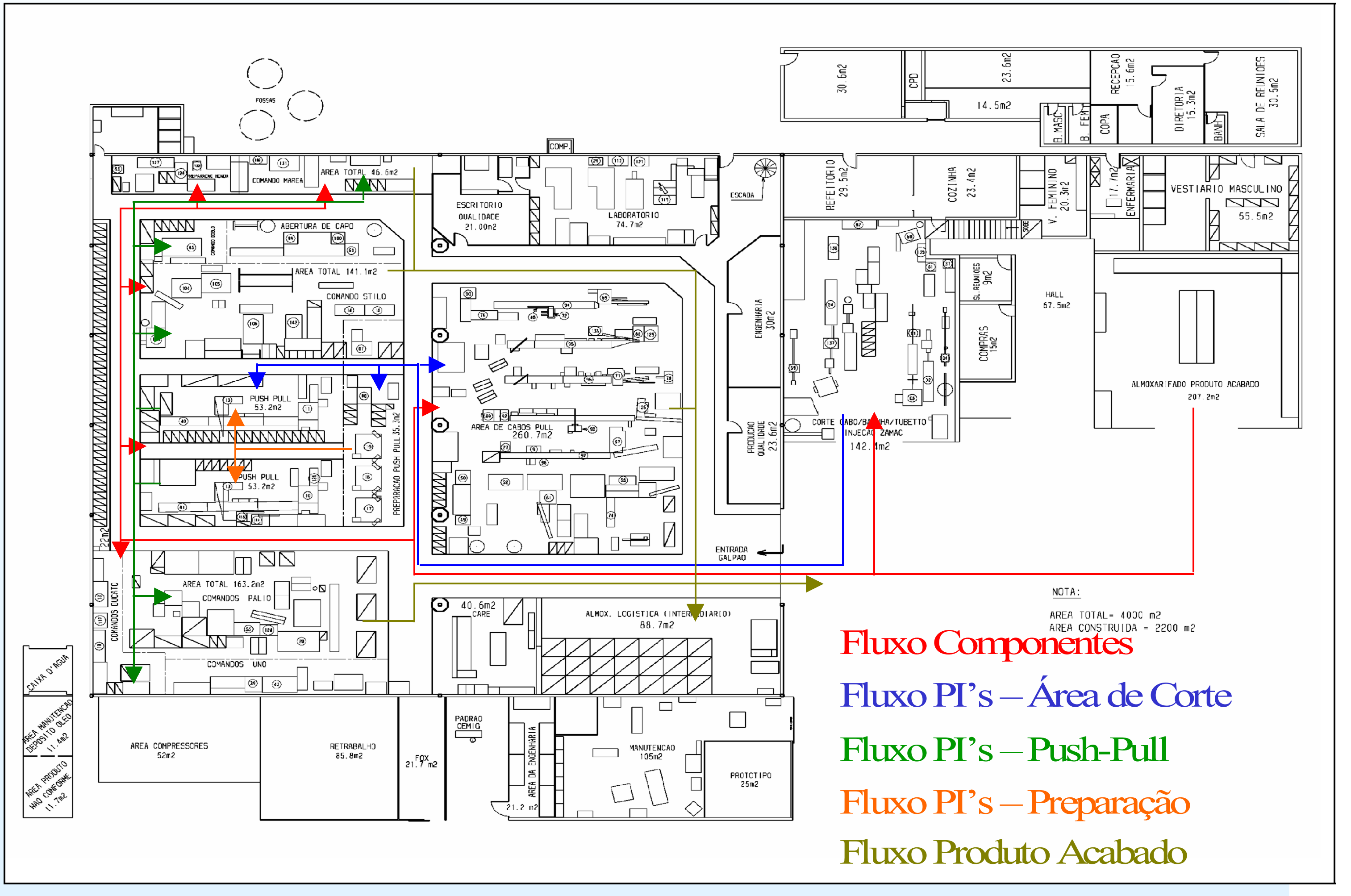
- His lay-out was organized in two major areas: docking area and cellular manufacturing area

Problem Description

SILA has 13 manufacturing cells:

- 1. Cutting
- 2. Push-Pull setup
- 3. Push-Pull shifts
- 4. Pull shifts
- 5. Shift Gears PALIO
- 6. Shift Gears UNO
- 7. Shift Gears STILO/IDEA
- 8. Shift Gears DOBLO
- 9. Shift Gears DUCATO
- 10. Shift Gears MAREA
- 11. Inflow area (docking)
- 12. Finished product area (docking)
- 13. Tubular elements area (docking)

The Original Lay-out and Production Flows



Assignment Costs

CUSTOS DE ALOCAÇÃO

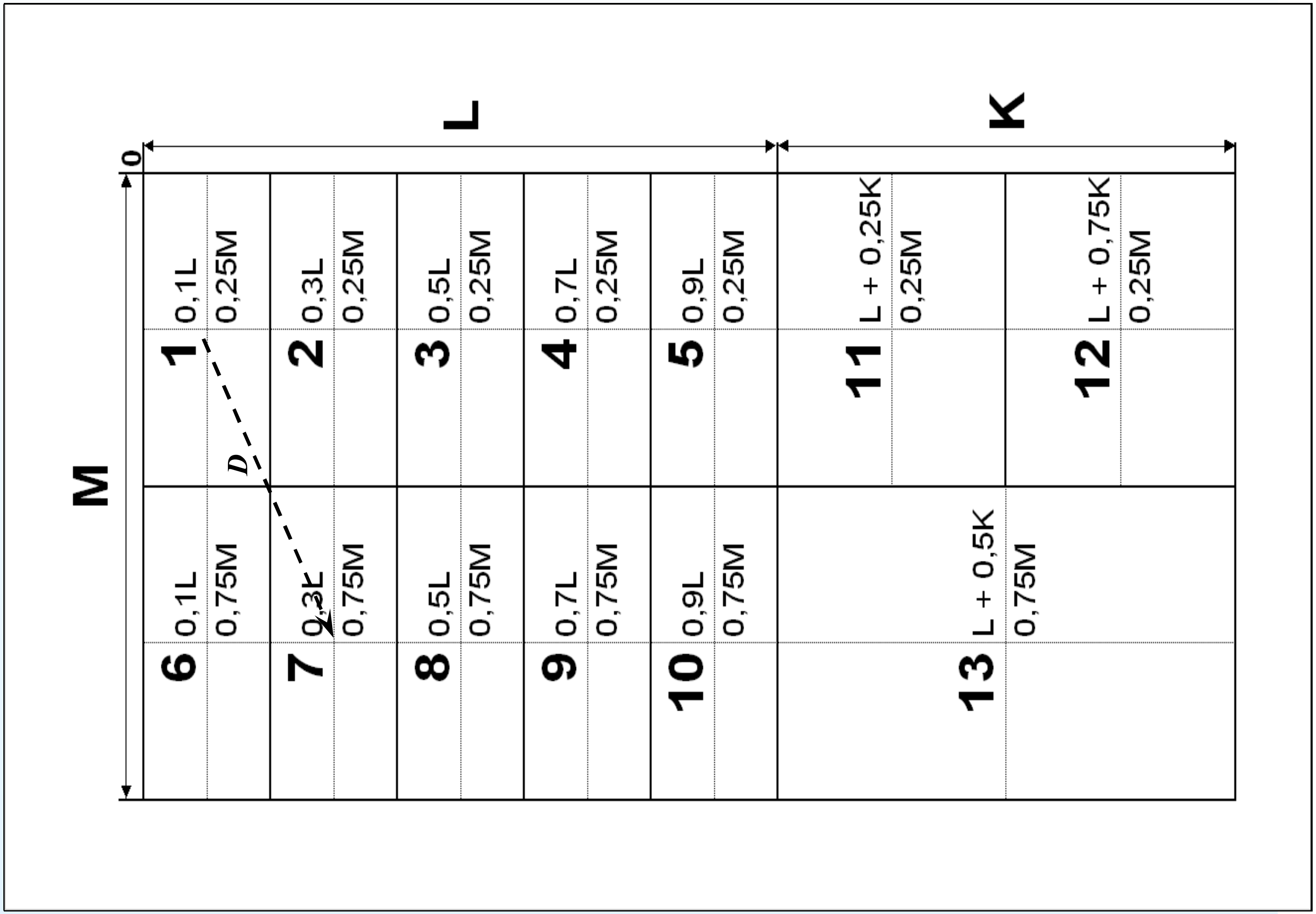
	ÁREA DE CORTE	PREPARAÇÃO PUSH-PULL E HONDA	CABOS PUSH-PULL	CABOS PULL	CC PALIO	CC UNO	CC STILO/IDEA	CC DOBLO	CC DUCATO	CC MAREA	RECEBIMENTO	ÁREA DE PRODUTO ACABADO	ÁREA DE TUBULARES
1	0	0	0	0	0	0	0	0	0	0	1000000	1000000	1000000
2	0	0	0	0	0	0	0	0	0	0	1000000	1000000	1000000
3	0	0	0	0	0	0	0	0	0	0	1000000	1000000	1000000
4	0	0	0	0	0	0	0	0	0	0	1000000	1000000	1000000
5	0	0	0	0	0	0	0	0	0	0	1000000	1000000	1000000
6	0	0	0	0	0	0	0	0	0	0	1000000	1000000	1000000
7	0	0	0	0	0	0	0	0	0	0	1000000	1000000	1000000
8	0	0	0	0	0	0	0	0	0	0	1000000	1000000	1000000
9	0	0	0	0	0	0	0	0	0	0	1000000	1000000	1000000
10	0	0	0	0	0	0	0	0	0	0	1000000	1000000	1000000
11	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	0	0	0
12	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	0	0	0
13	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	0	0	0

The Mass Flow Matrix

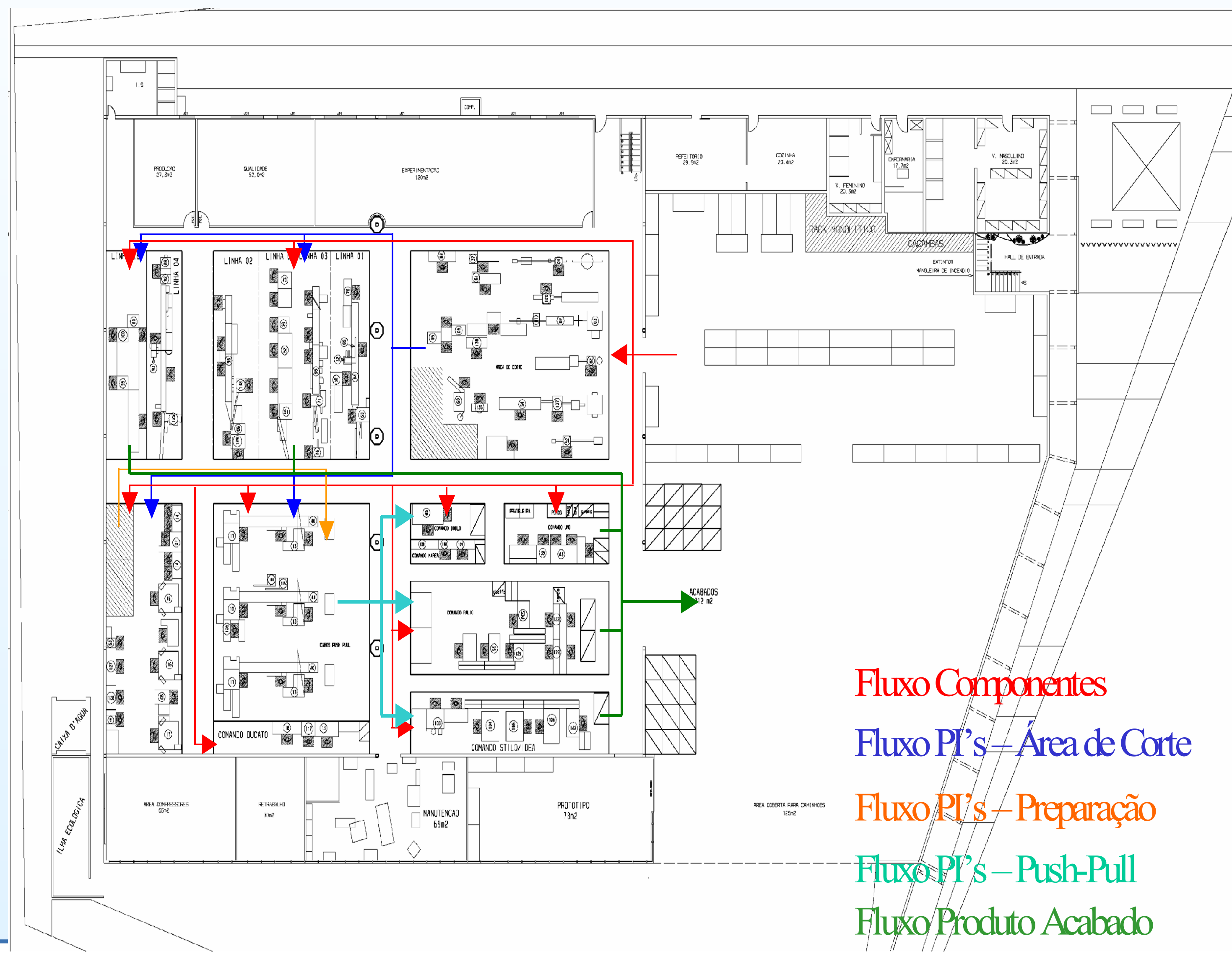
FLUXO DE MASSA ENTRE CÉLULAS

	ÁREA DE CORTE	PREPARAÇÃO PUSH-PULL E HONDA	CABOS PUSH-PULL	CABOS PULL	CC PALIO	CC UNO	CC STILO/IDEA	CC DOBLO	CC DUCATO	CC MAREA	RECEBIMENTO	ÁREA DE PRODUTO ACABADO	ÁREA DE TUBULARES
ÁREA DE CORTE	0	12375	24750	44300	0	0	0	0	0	0	0	0	0
PREPARAÇÃO PUSH-PULL E HONDA	0	0	42235	0	0	0	0	0	0	0	0	0	0
CABOS PUSH-PULL	0	0	0	0	34959	19004	5471	1569	730	267	0	2649	0
CABOS PULL	0	0	0	0	0	0	0	0	0	0	0	15574	0
CC PALIO	0	0	0	0	0	0	0	0	0	0	0	276685	0
CC UNO	0	0	0	0	0	0	0	0	0	0	0	83504	0
CC STILO/IDEA	0	0	0	0	0	0	0	0	0	0	0	45974	0
CC DOBLO	0	0	0	0	0	0	0	0	0	0	0	6194	0
CC DUCATO	0	0	0	0	0	0	0	0	0	0	0	2850	0
CC MAREA	0	0	0	0	0	0	0	0	0	0	0	1656	0
RECEBIMENTO	108000	40000	500	3856	60000	50000	10000	1000	800	800	0	0	0
ÁREA DE PRODUTO ACABADO	0	0	0	0	0	0	0	0	0	0	0	0	0
ÁREA DE TUBULARES	0	0	0	13343	202144	48060	34822	4357	1878	1091	0	0	0

The grid on the Shop Floor



The Final Lay-out and Production Flows



Achieved Improvements

- 12% of effective improvement of productivity.
- 15% of effective reduction of shop floor area.

Conclusions

Conclusions

- Miranda and Luna Fast Flow Formulation is more sensitive to the presence of linear terms.
- The impact of preprocessing is huge for Adams and Johnson Formulation.
- The use of a linear term appears to be a clue to break down the excessive symmetry of QAP.
- Interpreting the QAP as a network flow problem enables the solution of practical engineering problems.