Fast Flow Formulations for Sparse QAP Instances

ISMP 2006

Rio de Janeiro - Brasil

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Otimização Combinatória iacolor - p. 1/51

Quadratic Assignment Problems

Quadratic Assignment Problems

- Problem definition.
- Bounds and Linearizations.
- Fast Flow Formulations.
- Computational Experience.
- A Facility Lay-Out Problem.
- Conclusions

Quadratic Assignment Problems

Quadratic Assignment
 Problems

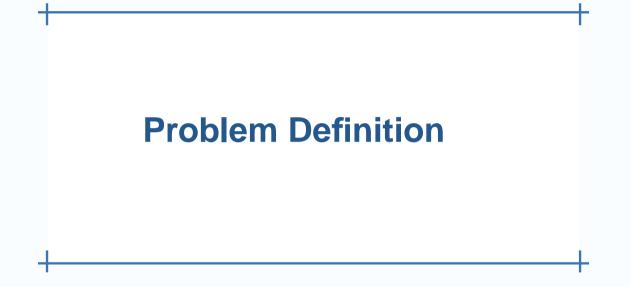
Problem Definition

Bounds and Linearizations

Fast Flow Formulations

Computational Experience

A Facility Lay-Out Problem



Motivation

Optimized resource allocation after World War II was a great challenge.

Quadratic Assignment Problems

Problem Definition

Motivation

- The Simplest Locational Model: Linear Assignment
- Linear Assignment
 Features
- The Great Idea
- Measuring Interdependency
- Quadratic Assignment Features

Bounds and Linearizations

Fast Flow Formulations

Computational Experience

A Facility Lay-Out Problem

Conclusions

• Cold War aspect: Captalism should be theoretically feasible.

• Koopmans and Beckmann: First work to devise and define the problem.

The Simplest Locational Model: Linear Assignment

Quadratic Assignment Problems	(1) $\max p = \sum_{k=1}^{n} \sum_{i=1}^{n} a_{ki} x_{ki}$
Problem Definition Motivation The Simplest Locational Model: Linear Assignment 	subject to: $k=1$ $i=1$
 Linear Assignment Features The Great Idea Measuring Interdependency Quadratic Assignment Features 	(2) $\sum_{k=1}^{n} x_{ki} = 1, \forall \ i = 1,, n$
Bounds and Linearizations	(3) $\sum_{i=1}^{n} x_{ki} = 1, \forall \ k = 1,, n$
Computational Experience A Facility Lay-Out Problem	(4) $x_{ki} \in \{0,1\}, \forall \ k, i = 1,, n.$

Linear Assignment Features

• Easy to read: His dual is the same as the Transportation Problem.

Quadratic Assignment Problems

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Bounds and Linearizations

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Computational Experience

A Facility Lay-Out Problem

- Easy to Solve: The Hungarian Method is a great tool for LAP.
- Drawback: Is too simple for real-world applications.

	The Great Idea
Quadratic Assignment Problems	
Problem Definition	
 Motivation The Simplest Locational 	
Model: Linear Assignment	QAP = LAP + Interdependency.
 Linear Assignment Features 	
The Great Idea	
 Measuring Interdependency 	
Quadratic Assignment	
Features	
Bounds and Linearizations	
Fast Flow Formulations	
Computational Experience	
A Facility Lay-Out Problem	

Conclusions

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Measuring Interdependency

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Quadratic Assignment Problems

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Conclusions

 $q = \sum \sum b_{kl} x_{ki} c_{ij} x_{lj}$ (k,l)(i,j)

Interdependency is evaluated by flows of intermediate commodities among different economic activities.

Quadratic Assignment Features

- Hard to read: It is not simple how to interpret his dual information.
- Hard to Solve: Strong formulations too hard.
- Hard to Solve: Easy formulations too weak.
- Hard to Solve: If $n \ge 20$ the instance is considered very hard.

Quadratic Assignment Problems

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Bounds and Linearizations

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Bounds and Linearizations

Quadratic Assignment Problems

Problem Definition

Bounds and Linearizations

- Bounds and Linearizations
- Koopmans and Beckmann
 Flow Formulation
- Gilmore and Lawler Bound
- Gilmore-Lawler Bound
- Lawler Formulation

 Frieze and Yadegar Formulation

- Adams and Johnson Formulation
- Important Remarks

Fast Flow Formulations

Computational Experience

A Facility Lay-Out Problem

- Koopmans and Beckmann Flow Formulation.
- Gilmore and Lawler Bound.
- Lawler formulation.
- Frieze and Yadegar Formulation.
- Adams and Johnson Formulation.

Koopmans and Beckmann Flow Formulation

Quadratic Assignment Problems (6)

Problem Definition

Bounds and Linearizations

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- Gilmore-Lawler Bound
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Fast Flow Formulations

Computational Experience

A Facility Lay-Out Problem

Conclusions

$$\max \sum_{k=1}^{n} \sum_{i=1}^{n} a_{ki} x_{ki} - \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} c_{ij} f_{ij}^{kl}$$

subject to (2) - (4) e:

$$(7) b_{kl} x_{ki} + \sum_{j=1}^{n} f_{ji}^{kl} = b_{kl} x_{li} + \sum_{j=1}^{n} f_{ij}^{kl}, \forall i, k, l = 1, ..., n$$

$$(8) \qquad f_{ii}^{kl} = 0, \forall i, k, l = 1, ..., n$$

$$(9) \qquad f_{ij}^{kl} \ge 0, \forall i, j, k, l = 1, ..., n$$

Drawback: Is too weak for the problem

Gilmore and Lawler Bound

Quadratic Assignment Problems

(10)

max
$$p = \sum_{k=1}^{n} \sum_{i=1}^{n} (a_{ki} - \lambda_{ki}) x_{ki}$$

Problem Definition

subject to:

(11)

(12)

(13)

Bounds and Linearizations

Bounds and Linearizations

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Fast Flow Formulations

Computational Experience

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Conclusions

 $\sum_{k=1}^{n} x_{ki} = 1, \quad \forall \ i = 1, ..., n$ $\sum_{i=1}^{n} x_{ki} = 1, \quad \forall \ k = 1, ..., n$ $x_{ki} \in \{0, 1\}, \quad \forall \ k, i = 1, ..., n.$

Gilmore-Lawler Bound

 \boldsymbol{n}

i=1

n

l=1

Quadratic Assignment Problems

$$\lambda_{ki} = \max \sum_{l=1}^{n} \sum_{j=1}^{n} b_{kl} c_{ij} x_{lj}$$

 $\sum x_{lj} = 1, \forall l = 1, ..., n,$

Problem Definition

(14)

(15)

(16)

(17)

(18)

Bounds and Linearizations
 Bounds and Linearizations

- Koopmans and Beckmann Flow Formulation
- Gilmore and Lawler Bound
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- Fast Flow Formulations

Computational Experience

A Facility Lay-Out Problem

 $x_{ki} = 1,$ $x_{lj} \ge 0, \forall l, j = 1, ..., n.$

 $x_{lj} = 1, \forall j = 1, ..., n,$

Conclusions

GLB takes $n^2 + 1$ LAP solutions. It is tight for $n \le 20$.

Lawler Formulation

 \boldsymbol{n}

n

Quadratic Assignment Problems

(1

(23)

Problem Definition

Bounds and Linearizations

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Fast Flow Formulations

Computational Experience

A Facility Lay-Out Problem

Conclusions

(19)
$$\max \sum_{k=1}^{n} \sum_{i=1}^{n} a_{ki} x_{ki} - \sum_{k=1}^{n} \sum_{l=1}^{n} \sum_{j=1}^{n} d_{kilj} y_{kilj}$$

subject to (2) - (4) e:
(20)
$$\sum_{k,l,i,j=1}^{n} y_{kilj} = n^{2}$$

(21)
$$y_{kilj} \leq x_{ki}, \forall i, j, k, l = 1, ..., n, i \neq j, k \neq l$$

(22)
$$y_{kilj} \leq x_{lj}, \forall i, j, k, l = 1, ..., n, i \neq j, k \neq l$$

 \boldsymbol{n}

n

n

 \boldsymbol{n}

$$y_{kilj} \geq 0, \ \forall i, j, k, l = 1, ..., n, \ i \neq j, \ k \neq l$$

The central idea: $x_{ki}x_{lj} = y_{kilj}$. Here $d_{ijkl} = b_{kl}c_{ij}$. Too large: n^4 variables and n^4 linking constraints.

Frieze and Yadegar Formulation

Quadratic Assignment Problems Problem Definition	(24) $\max \sum_{k=1}^{n} \sum_{i=1}^{n} a_{ki} x_{ki} - \sum_{k=1}^{n} \sum_{l=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} d_{kilj} y_{kilj}$
Bounds and Linearizations	subject to (2) - (4) e:
 Bounds and Linearizations Bounds and Linearizations Koopmans and Beckmann Flow Formulation Gilmore and Lawler Bound Gilmore-Lawler Bound 	(25) $\sum_{k=1}^{n} y_{kilj} = x_{ki}, \ \forall i, k, l = 1,, n, \ i \neq j, \ k \neq l$
 Clinicle-Lawler Bound Lawler Formulation Frieze and Yadegar Formulation Adams and Johnson Formulation Important Remarks 	(26) $\sum_{l=1}^{n} y_{kilj} = x_{ki}, \ \forall i, j, k = 1,, n, \ i \neq j, \ k \neq l$
Fast Flow Formulations Computational Experience	(27) $\sum_{k=1}^{n} y_{kilj} = x_{lj}, \ \forall i, j, l = 1,, n, \ i \neq j, \ k \neq l$
A Facility Lay-Out Problem	k = 1 n
Conclusions	(28) $\sum y_{kilj} = x_{lj}, \ \forall j, l, k = 1,, n, \ i \neq j, \ k \neq l$
	(29) $i=1$ $y_{kilj} \geq 0, \forall i, j, k, l = 1,, n, i \neq j, k \neq l$
	Another big model: n^4 variables and $4n^3$ linking constraints.

Adams and Johnson Formulation

Quadratic Assignment Problems (3

 \boldsymbol{n}

n

Problem Definition

Bounds and Linearizations

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Fast Flow Formulations

Computational Experience

A Facility Lay-Out Problem

Conclusions

$$0) \max \sum_{k=1}^{n} \sum_{i=1}^{n} a_{ki} x_{ki} - \sum_{k=1}^{n} \sum_{l=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} d_{kilj} y_{kilj}$$

subject to (2) - (4) e:

(31)
$$\sum_{j=1}^{n} y_{kilj} = x_{ki}, \ \forall i, k, l = 1, ..., n, \ i \neq j, \ k \neq l$$

(32)
$$\sum_{l=1} y_{kilj} = x_{ki}, \forall i, j, k = 1, ..., n, i \neq j, k \neq l$$

33)
$$y_{kilj} = y_{ljki}, \forall i, j, k, l = 1, ..., n, i \neq j, k \neq l,$$

34) $y_{kilj} \ge 0, \forall i, j, k, l = 1, ..., n, i \neq j, k \neq l$

Yet another large formulation: n^4 variables and $n^4 + 2n^3$ linking constraints. As tight as Frieze and Yadegar formulation until we are

As tight as Frieze and Yadegar formulation until we are aware.

Important Remarks

Quadratic Assignment Problems

Problem Definition

Bounds and Linearizations

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Fast Flow Formulations

Computational Experience

A Facility Lay-Out Problem

Conclusions

- The main difficulty with QAP appears to be the excessive symmetry.
- A good alternative to avoid this symmetry is to include linear coefficients.
- It is necessary to find a formulation that balances bound quality and computational effort.

What if we could find a formulation that explores the non null linear term in the best way? What if this formulation combines a bound not so poor being also easy to solve?

Fast Flow Formulations

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Fast Flow Formulations

- Miranda and Luna Enhanced Flow Formulation.
- Preprocessing for CPLEX.
- Improving Bounds.
- Re-interpreting Adams and Johnson.

Quadratic Assignment Problems

Problem Definition

Bounds and Linearizations

Fast Flow Formulations

- Fast Flow Formulations
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Computational Experience

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Miranda and Luna: Enhanced Flow Formulation

Quadratic Assignment Problems

Problem Definition

Bounds and Linearizations

Fast Flow Formulations

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Computational Experience

A Facility Lay-Out Problem

(35)		$\sum_{n=1}^{n} \sum_{i=1}^{n}$	$\sum a_{ki} x_{ki} - \sum \sum c_{ij} f_{ij}^{kl}$
	subject	to (2	2) - (4) and:
(36)–	$\sum_{j=1}^{n} f_{ij}^{kl}$	=	$-b_{kl}x_{ki}$, $\forall i, k, l = 1,, n, i \neq j, k \neq l$
(37)	$\sum_{i=1}^{n} f_{ij}^{kl}$	=	$b_{kl}x_{lj}$, $\forall j, k, l = 1,, n, i \neq j, k \neq l$
(38)	$b_{lk} f_{ij}^{kl}$	=	$b_{kl}f_{ji}^{lk}$, $\forall i, j, k, l = 1,, n, i \neq j, k \neq l$
(39)	f_{ij}^{kl}	\geq	$0 \ , \ \forall \ i,j,k,l=1,,n, i \neq j \ , k \neq l$

Preprocessing for CPLEX

Quadratic Assignment Problems

Problem Definition

Bounds and Linearizations

Fast Flow Formulations

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Computational Experience

A Facility Lay-Out Problem

Conclusions

$$(40)\max \sum_{k=1}^{n} \sum_{i=1}^{n} a_{ki}x_{ki} - \sum_{i \neq j} \sum_{(k,l) \in \phi} [c_{ij}b_{kl} + c_{ji}b_{lk}]f_{ij}^{kl}$$

subject to (2) - (4) and:

$$(41) \sum_{j \neq i} f_{ij}^{kl} = x_{ki}, \forall i = 1, ..., n, \forall (k,l) \in \phi$$

$$(42) \sum_{i \neq j} f_{ij}^{kl} = x_{lj}, \forall j = 1, ..., n, \forall (k,l) \in \phi$$

$$(43) \qquad f_{ij}^{kl} \geq 0, \forall i, j = 1, ..., n, i \neq j, \forall (k,l) \in \phi$$

where:

(44) $\phi = \{ (k,l) | k < l \land (b_{kl} \neq 0 \lor b_{lk} \neq 0) \}$

Improving Bounds

Quadratic Assignment Problems

Problem Definition

Bounds and Linearizations

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A Facility Lay-Out Problem

Conclusions

If the bound is too poor in a given instance, we can add two families of valid inequalities. These inequalities are transliterations of Frieze and Yadegar work.

$$(45) \sum_{k, (k,l) \in \phi} f_{ij}^{kl} \leq x_{lj}, \forall i, j = 1, ..., n, i \neq j, \forall l, (k,l) \in \phi$$
$$\sum_{l, (k,l) \in \phi} f_{ij}^{kl} \leq x_{ki}, \forall i, j = 1, ..., n, i \neq j, \forall k, (k,l) \in \phi$$

Re-interpreting Adams and Johnson

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Computational Experience

A Facility Lay-Out Problem

(48)

(49)

(50)

Conclusions

It is possible to re-interpret Adams and Johnson formulation as a flow formulation. The idea is to take advantage of sparsity without compromising LP bounds.

(46) max $\sum \sum a_{ki} x_{ki} - \sum \sum [c_{ij} b_{kl} + c_{ji} b_{lk}] f_{ij}^{kl}$ $k = 1 \ i = 1$ $i \neq j \quad (k,l) \in \phi$ subject to (2) - (4) and: (47) $\sum f_{ij}^{kl} = x_{ki}, \forall i = 1, ..., n, \forall (k, l) \in \phi$ $i \neq i$ $\sum f_{ij}^{kl} = x_{lj} , \forall j = 1, ..., n, \forall (k,l) \in \phi$ $i \neq j$ $\sum f_{ij}^{kl} + \sum f_{ji}^{lk} \leq x_{ki}, \forall k \in K, \forall i \neq j$ $l \in \phi | k < l$ $l \in \phi | k > l$ $f_{ij}^{kl} \geq 0 , \, \forall \, i, j = 1, ..., n, i \neq j \,, \, \forall (k,l) \in \phi$

Computational Experience

Comparisons

Quadratic Assignment Problems

Problem Definition

Bounds and Linearizations

Fast Flow Formulations

Computational Experience

- Comparisons
- LP Bounds: AJ94 x ML04
- Computing Times (LP): AJ94 x ML04.
- Computing Times (MIP): AJ94 x ML04.
- LP Bounds: GLB x ML04
- LP Bounds: GLB x ML04
- Computing Times (MIP): GLB x ML04.
- After Preprocessing
- Tempos de Computação (MIP): GLB x CML04
- After Improving Bounds
- Limites de PL: GLB x SCML04
- Preprocessing Adams and Johnson
- Limites de PL: [AJ94] Versus [CRLT-level1]

A Facility Lay-Out Problem

- LP Bounds: AJ94 x ML04.
- Computing Times: AJ94 x ML04.
- LP Bounds: GLB x ML04.
- Computing times: GLB x ML04.

LP Bounds: AJ94 x ML04

Quadratic Assignment Problems

Problem Definition

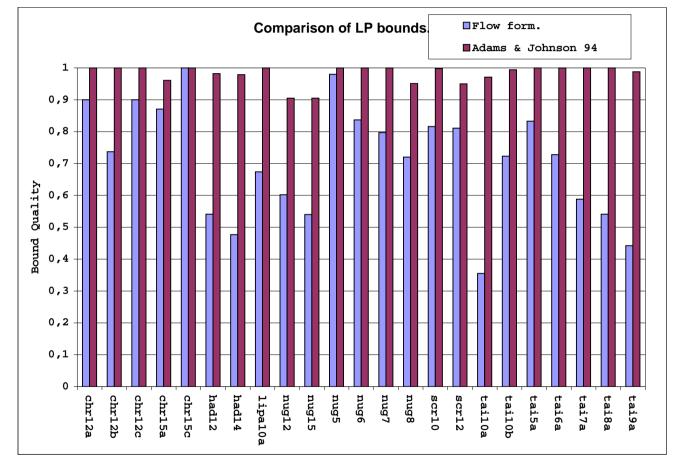
Bounds and Linearizations

Fast Flow Formulations

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A Facility Lay-Out Problem



Computing Times (LP): AJ94 x ML04.

Quadratic Assignment Problems

Problem Definition

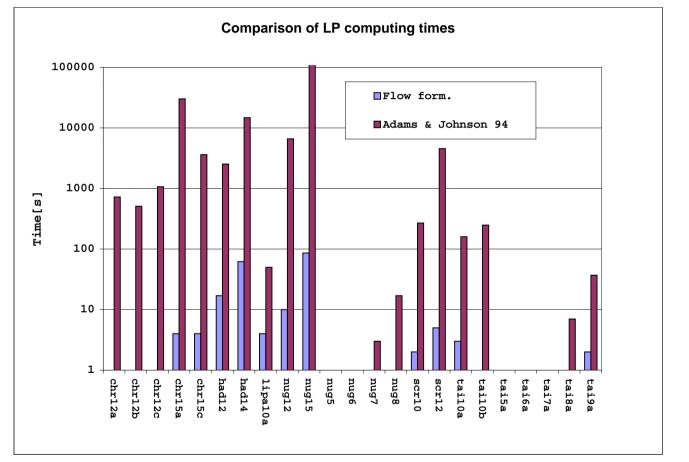
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A Facility Lay-Out Problem



Computing Times (MIP): AJ94 x ML04.

	Original	Problem	Variables		p/q	Flow form.	Adams and Johnson
Quadratic Assignment	instance	size	Integer	Continuous	ratio	time[s]	time[s]
Problems	mchr12a				0.426	4	584
Problem Definition	mchr12b	12	144	17424	0.383	5	516
	mchr12c				0.314	3	803
Bounds and Linearizations	mchr15a				0.631	16	14209
	mchr15a	15	225	44100	0.856	6	10125
Fast Flow Formulations	mchr15c				0.807	9	3058
Computational Experience	mchr18a				0.564	116	*
ComparisonsLP Bounds: AJ94 x ML04	mchr18b	18	324	93636	1.864	7	*
	mchr20a				1.047	76	*
 Computing Times (LP): AJ94 x ML04. 	mchr20b	20	400	144400	0.824	47	*
• Computing Times (MIP):	mchr20c				1.056	108	*
AJ94 x ML04. • LP Bounds: GLB x ML04	mchr22a	22	484	213444	0.304	138	*
 LP Bounds: GLB x ML04 Computing Times (MIP): 	mchr22b				0.293	96	*

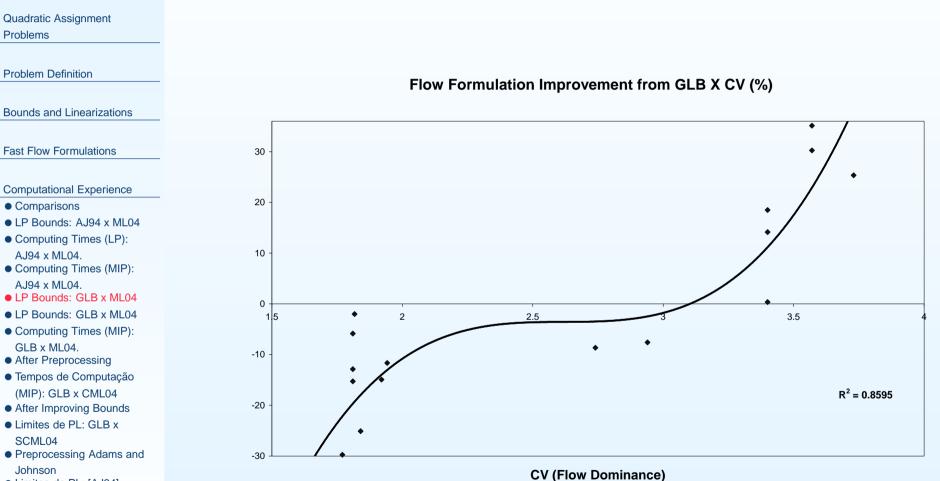
- (MIP): GLB x CML04 • After Improving Bounds • Limites de PL: GLB x
- SCML04

GLB x ML04. After Preprocessing • Tempos de Computação

- Preprocessing Adams and Johnson
- Limites de PL: [AJ94] Versus [CRLT-level1]

A Facility Lay-Out Problem

LP Bounds: GLB x ML04



• Limites de PL: [AJ94] Versus [CRLT-level1]

A Facility Lay-Out Problem

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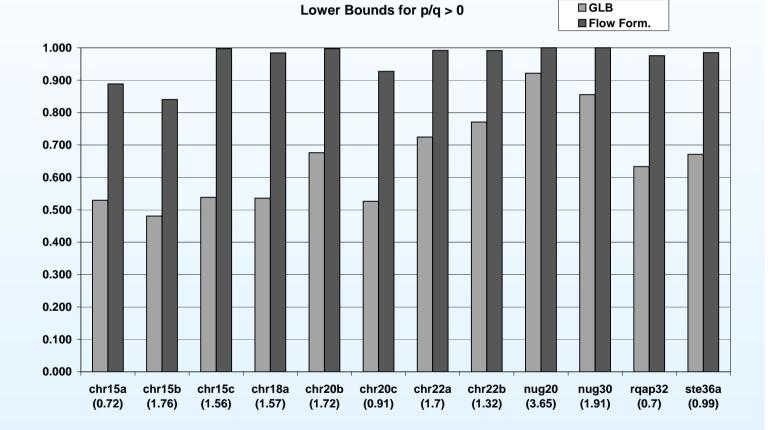
Bounds and Linearizations

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 Versus [CRLT-level1]

A Facility Lay-Out Problem



Computing Times (MIP): GLB x ML04.

	Original	Instance	Variables		p/q	Flow form.	Branch-and-Bound
Quadratic Assignment	Instance	size	Integer	Continuous	ratio	time[s]	(GLB) time[s]
Problems	mchr15b	15	225	44100	0.726	36	0
Problem Definition	mchr18a	18	324	93636	1.304	41	26
	mchr20c	20	400	144400	0.707	54	75
Bounds and Linearizations	mchr22a	22	484	213444	0.744	19	11
	mnug20	20	400	144400	3.268	42	0
Fast Flow Formulations	mnug30	30	900	756900	1.358	1779	23
Computational Experience	rqap32	32	1024	984064	0.701	1964	2503
Comparisons	rqap33	33	1089	1115136	0.97	3236	3060
• LP Bounds: AJ94 x ML04	rqap34	34	1156	1258884	0.74	7207	7198
 Computing Times (LP): AJ94 x ML04. 	rqap35	35	1225	1416100	0.87	2610	16154
• Computing Times (MIP):	mste36a				0.687	2457	27974
AJ94 x ML04. • LP Bounds: GLB x ML04	mste36a	36	1296	1587600	1.199	2801	2925
• LP Bounds: GLB x ML04	mtho40				0.741	10532	*
 Computing Times (MIP): GLB x ML04. After Preprocessing 	mtho40	40	1600	2433600	1.040	7689	1984

- Tempos de Computação (MIP): GLB x CML04
- After Improving Bounds
- Limites de PL: GLB x SCML04
- Preprocessing Adams and Johnson
- Limites de PL: [AJ94] Versus [CRLT-level1]

A Facility Lay-Out Problem

Conclusions

After Preprocessing

Quadratic Assignment Problems

Problem Definition

Bounds and Linearizations

Fast Flow Formulations

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- Comparisons
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A Facility Lay-Out Problem

After Preprocessing:

Tempos de Computação (MIP): GLB x CML04

	Original	Instance	p/q	Flow Form.	Branch-And-Bound	Compact Flow
Quadratic Assignment	Instance	size	ratio	Time [s]	(GLB) Time [s]	Form. Time [s]
Problems	mchr15a		0.631	16	0	14
	mchr15b	15	0.726	36	0	9
Problem Definition	mchr15c		0.807	9	0	8
Bounds and Linearizations	mchr18a		0.564	116	83	74
	mchr18a	18	1.304	41	26	22
Fast Flow Formulations	mchr20b		1.702	10	15	13
Computational Experience	mchr20c	20	0.707	54	75	36
Comparisons	mchr22a		0.744	19	11	2
• LP Bounds: AJ94 x ML04	mchr22b	22	1.341	18	4	6
 Computing Times (LP): AJ94 x ML04. 	mnug20	20	3.268	42	0	4
 Computing Times (MIP): 		20		80		
AJ94 x ML04. ● LP Bounds: GLB x ML04	mnug20		4.477		0	4
 LP Bounds: GLB x ML04 LP Bounds: GLB x ML04 	mnug30	30	1.358	1779	23	200
Computing Times (MIP):	rqap32	32	0.701	1964	2503	1306
GLB x ML04. • After Preprocessing	rqap33	33	0.97	3236	3060	1735
 Tempos de Computação 	rqap34	34	0.74	7207	7198	3417
(MIP): GLB x CML04	rqap35	35	0.87	2610	16154	1956
 After Improving Bounds Limites de PL: GLB x 	mste36a	36	1.199	2801	2925	325
SCML04						
 Preprocessing Adams and 	mtho40		0.741	10532	*	5362
Johnson ● Limites de PL: [AJ94]	mtho40	40	1.040	7689	1984	1551
 Limites de PL: [AJ94] Versus [CRLT-level1] 	msko42		0.854	*	*	16676
	msko42	42	1.280	*	*	3724
A Facility Lay-Out Problem		.2				0.21

After Improving Bounds

Quadratic Assignment	
Problems	

Problem Definition

Bounds and Linearizations

Fast Flow Formulations

Computational Experience

- Comparisons
- LP Bounds: AJ94 x ML04
- Computing Times (LP): AJ94 x ML04.
- Computing Times (MIP): AJ94 x ML04.
- LP Bounds: GLB x ML04
- LP Bounds: GLB x ML04
- Computing Times (MIP): GLB x ML04.
- After Preprocessing
- Tempos de Computação (MIP): GLB x CML04
- After Improving Bounds
- Limites de PL: GLB x SCML04
- Preprocessing Adams and Johnson
- Limites de PL: [AJ94] Versus [CRLT-level1]

A Facility Lay-Out Problem

After Improving Bounds:

Limites de PL: GLB x SCML04

		MLC)4	Bound	SCML04		Bound	AJS	Bound	
Quadratic Assignment	instance	lp bound	time[s]	Quality	lp bound	time[s]	Quality	lp bound	time[s]	Quality
Problems	chr12a.dat	8593.12	1	0.900	9246.67	1	0.968	9552	725	1.000
Droblem Definition	chr12b.dat	7184	1	0.737	9742	1	1.000	9742	508	1.000
Problem Definition	chr12c.dat	10042.7	1	0.900	10121.1	1	0.907	11156	1068	1.000
Bounds and Linearizations	chr15a.dat	8621.94	4	0.871	9295.33	4	0.939	9513	30146	0.961
	chr15c.dat	9504	4	1.000	9504	4	1.000	9504	3622	1.000
Fast Flow Formulations	had12.dat	894	17	0.541	1543.78	1414	0.934	1621.54	2533	0.982
Computational Experience	had14.dat	1300.5	62	0.477	2583.07	9269	0.948	2666.12	14778	0.979
Computational Experience • Comparisons	lipa10a.dat	318.8	4	0.674	448.123	161	0.947	473	50	1.000
• LP Bounds: AJ94 x ML04	nug12.dat	348	10	0.602	471.821	371	0.816	522.89	6597	0.905
 Computing Times (LP): AJ94 x ML04. 	nug15.dat	621	86	0.540	912.063	8790	0.793	1041	131923	0.905
 Computing Times (MIP): 	nug5.dat	49	1	0.980	50	1	1.000	50	1	1.000
AJ94 x ML04. • LP Bounds: GLB x ML04	nug6.dat	72	1	0.837	80	1	0.930	86	1	1.000
• LP Bounds: GLB x ML04	nug7.dat	118	1	0.797	134.438	1	0.908	148	3	1.000
 Computing Times (MIP): GLB x ML04. 	nug8.dat	154	1	0.720	191	3	0.893	203.5	17	0.951
After Preprocessing	scr10.dat	21958	2	0.816	24245.8	9	0.901	26873.1	269	0.998
 Tempos de Computação (MIP): GLB x CML04 	scr12.dat	25474	5	0.811	27368.7	42	0.871	29827.3	4555	0.950
After Improving Bounds	tai10a.dat	47953.3	3	0.355	112221	70	0.831	131098	160	0.971
 Limites de PL: GLB x SCML04 	tai10b.dat	855788	1	0.723	1139856.99	30	0.963	1176140	248	0.994
 Preprocessing Adams and 	tai5a.dat	10747	1	0.833	12902	1	1.000	12902	1	1.000
Johnson • Limites de PL: [AJ94]	tai6a.dat	21427.8	1	0.728	28013.4	1	0.952	29432	1	1.000
Versus [CRLT-level1]	tai7a.dat	31730.1	1	0.588	47948	1	0.888	53976	1	1.000
A Facility Lay-Out Problem	tai8a.dat	41952.2	1	0.541	69705.1	1	0.899	77502	7	1.000
	tai9a.dat	41816	2	0.442	83180.9	24	0.879	93501	37	0.988
Operations										

Conclusions

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Preprocessing Adams and Johnson

Quadratic Assignment Problems

Problem Definition

Bounds and Linearizations

Fast Flow Formulations

Computational Experience

- Comparisons
- LP Bounds: AJ94 x ML04
- Computing Times (LP): AJ94 x ML04.
- Computing Times (MIP): AJ94 x ML04.

• LP Bounds: GLB x ML04

- LP Bounds: GLB x ML04
- Computing Times (MIP): GLB x ML04.
- After Preprocessing
- Tempos de Computação (MIP): GLB x CML04
- After Improving Bounds
- Limites de PL: GLB x SCML04
- Preprocessing Adams and Johnson
- Limites de PL: [AJ94] Versus [CRLT-level1]

A Facility Lay-Out Problem

Preprocessing AJ94:

Limites de PL: [AJ94] Versus [CRLT-level1]

	Instar	ice	Co	mpact RLT le	vel1	F	RLT level 1 [AJ	94]
Quadratic Assignment	name	size	lp bound	time [s]	bound quality	lp bound	time [s]	bound quality
Problems	nug5	5	50.0	0	1.0000	50.0	1	1.0000
Dashlara Dafallian	tai5a	5	12,902.0	0	1.0000	12,902.0	1	1.0000
Problem Definition	nug6	6	86.0	0	1.0000	86.0	1	1.0000
Bounds and Linearizations	tai6a	6	29,432.0	1	1.0000	29,432.0	1	1.0000
	nug7	7	148.0	1	1.0000	148.0	3	1.0000
Fast Flow Formulations	tai7a	7	53,976.0	1	1.0000	53,976.0	1	1.0000
	nug8	8	202.6	6	0.9469	203.5	17	0.9509
Computational Experience Comparisons	tai8a	8	77,502.0	5	1.0000	77,502.0	7	1.0000
LP Bounds: AJ94 x ML04	tai9a	9	93,501.0	26	0.9882	93,501.0	37	0.9882
Computing Times (LP):	lipa10a	10	473.0	50	1.0000	47 3.0	50	1.0000
AJ94 x ML04. • Computing Times (MIP):	scr10	10	26,659.0	12	0.9902	26,873.1	269	0.9982
AJ94 x ML04. ● LP Bounds: GLB x ML04	tai10a	10	131,053.0	86	0.9706	131,098.0	160	0.9709
• LP Bounds: GLB x ML04	tai10b	10	1,172,043.4	47	0.9901	1,176,140.0	248	0.9936
• Computing Times (MIP):	chr12a	12	9,552.0	2	1.0000	9,552.0	725	1.0000
GLB x ML04. • After Preprocessing	chr12b	12	9,332.0	2	1.0000		508	1.0000
 Tempos de Computação 				-		9,742.0		
(MIP): GLB x CML04 • After Improving Bounds	chr12c	12	10,927.3	1	0.9795	11,156.0	1,068	1.0000
• Limites de PL: GLB x	had12	12	1,621.5	1,633	0.9816	1,621.5	2,533	0.9816
SCML04 • Preprocessing Adams and	nug12	12	522.7	308	0.9043	522.9	6,597	0.9047
Johnson	scr12	12	29,766.5	59	0.9477	29,827.3	4,555	0.9496
Limites de PL: [AJ94]	had14	14	2,666.1	12,044	0.9788	2,666.1	14,778	0.9788
Versus [CRLT-level1]	chr15a	15	9,456.5	3	0.9556	9,513.0	30,146	0.9613
A Facility Lay-Out Problem	chr15c	15	9,504.0	2	1.0000	9,504.0	3,622	1.0000
	nug15	15	1,040.3	7,895	0.9046	1,041.0	131,923	0.9052
Conclusions	nug20	20	2,180.4	157,424	0.8484	2,181.6	2,310,985	0.8489

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A Facility Lay-Out Problem

Problem Description

• SILA BRASIL is an automotive components manufacturer in Belo Horizonte, Minas Gerais, Brasil.

- SILA is a part of FIAT S.A. supply chain.
- Its expertise is the manufacturing of precision components for 16-valve engines and other machine elements.

Quadratic Assignment Problems

Problem Definition

Bounds and Linearizations

Fast Flow Formulations

Computational Experience

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A Facility Lay-Out Problem
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- Problem Description
- Problem Description
- Problem Description
- The Original Lay-out and Production Flows
- Assignment Costs
- The Mass Flow Matrix
- The grid on the Shop Floor
- The Final Lay-out and Production Flows
- Achieved Improvements

Problem Description

Quadratic Assignment Problems

Problem Definition

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A Facility Lay-Out Problem

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Conclusions

• The main problem in its former lay-out was the excessive traffic of vehicles inside his plant to transport raw materials.

His lay-out was organized in two major areas: docking area and cellular manufacturing area

Problem Description

SILA has 13 manufacturing cells:

Quadratic Assignment Problems

- Problem Definition
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- Fast Flow Formulations
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- A Facility Lay-Out Problem
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- 1. Cutting
- 2. Push-Pull setup
- 3. Push-Pull shifts
- 4. Pull shifts
- 5. Shift Gears PALIO
- 6. Shift Gears UNO
- 7. Shift Gears STILO/IDEA
- 8. Shift Gears DOBLO
- 9. Shift Gears DUCATO
- 10. Shift Gears MAREA
- 11. Inflow area (docking)
- 12. Finished product area (docking)
- 13. Tubular elements area (docking)

The Original Lay-out and Production Flows

Quadratic Assignment Problems

Problem Definition

Bounds and Linearizations

Fast Flow Formulations

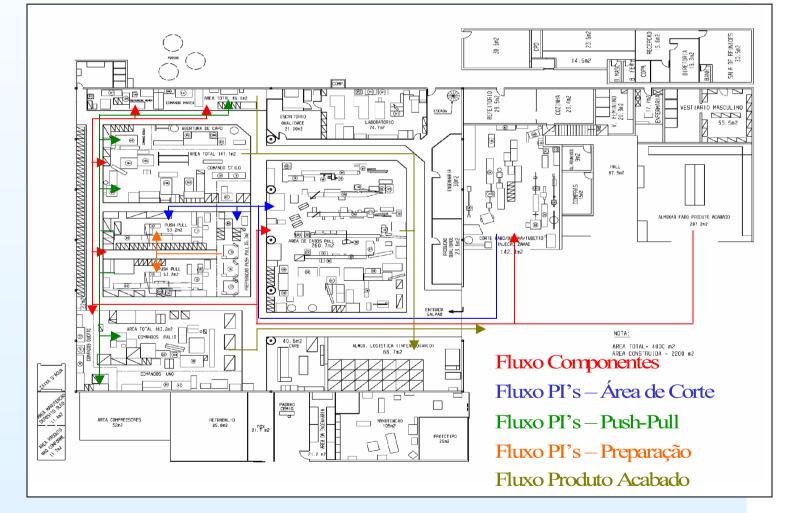
Computational Experience

A Facility Lay-Out Problem

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Assignment Costs

Quadratic Assignment Pr

Problems	CUSTOS DE ALOCAÇÃO															
Problem Definition			ÁREA DE CORTE	PREPARAÇÃO PUSH-PULL E HONDA	CABOS PUSH- PULL	CABOS PULL	CC PALIO	CC UNO	CC STILO/IDEA	CC DOBLO	CC DUCATO	CC MAREA	RECEBIMENTO	ÁREA DE PRODUTO ACABADO	ÁREA DE TUBULARES	
Bounds and Linearizations		1	0	0	0	0	0	0	0	0	0	0	1000000	1000000	1000000	
Fast Flow Formulations		2	0	0	0	0	0	0	0	0	0	0	1000000	1000000	1000000	
Computational Experience		3	0	0	0	0	0	0	0	0	0	0	1000000	1000000	1000000	
A Facility Lay-Out Problem		4	0	0	0	0	0	О	0	0	0	0	1000000	1000000	1000000	
 Problem Description Problem Description Problem Description The Original Lay-out and 		5	0	0	0	0	0	0	0	0	0	0	1000000	1000000	1000000	
			6	0	0	0	0	0	0	0	0	0	0	1000000	1000000	1000000
Production Flows Assignment Costs The Mass Flow Matrix 			7	0	0	0	0	0	0	0	0	0	0	1000000	1000000	1000000
 The grid on the Shop Floor The Final Lay-out and 			8	0	0	0	0	0	0	0	0	0	0	1000000	1000000	1000000
Production Flows • Achieved Improvements				9	0	0	0	0	0	0	0	0	0	0	1000000	1000000
Conclusions		10	0	0	0	0	0	0	0	0	0	0	1000000	1000000	1000000	
		11	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	0	0	0	
		12	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	0	0	0	
		13	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	0	0	0	

The Mass Flow Matrix

Quadratic Assignment Probler

Problems	FLUXO DE MASSA ENTRE CÉLULAS														
Problem Definition			ÁREA DE CORTE	PREPARAÇÃO PUSH-PULL E HONDA	CABOS PUSH- PULL	CABOS PULL	CC PALIO	CC UNO	CC STILO/IDEA	CC DOBLO	CC DUCATO	CC MAREA	RECEBIMENTO	ÁREA DE PRODUTO ACABADO	ÁREA DE TUBULARES
Bounds and Linearizations	_	ÁREA DE CORTE	0	12375	24750	44300	0	0	0	0	0	0	0	0	0
Fast Flow Formulations		PREPARAÇÃO PUSH-PULL E HONDA	0	0	42235	0	0	0	0	0	0	0	0	0	0
Computational Experience		CABOS PUSH- PULL	0	0	0	0	34959	19004	5471	1569	730	267	0	2649	0
A Facility Lay-Out Problem		CABOS PULL	0	0	0	0	0	0	0	0	0	0	0	15574	0
 Problem Description Problem Description 		CC PALIO	0	0	0	0	0	0	0	0	0	0	0	276685	0
 Problem Description The Original Lay-out and 	-	CC UNO	0	0	0	0	0	0	0	0	0	0	0	83504	0
 Production Flows Assignment Costs The Mass Flow Matrix 	-	CC STILO/IDEA	0	0	0	0	0	0	0	0	0	0	0	45974	0
 The grid on the Shop Floor The Final Lay-out and 	-	CC DOBLO	0	0	0	0	0	0	0	0	0	0	0	6194	0
Production Flows • Achieved Improvements	-	CC DUCATO	0	0	0	0	0	0	0	0	0	0	0	2850	0
Conclusions	-	CC MAREA	0	0	0	0	0	0	0	0	0	0	0	1656	0
		RECEBIMENTO	108000	40000	500	3856	60000	50000	10000	1000	800	800	0	0	0
		ÁREA DE PRODUTO ACABADO	0	0	0	0	0	0	0	0	0	0	0	0	0
		ÁREA DE TUBULARES	0	0	0	13343	202144	48060	34822	4357	1878	1091	0	0	0

The grid on the Shop Floor

Quadratic Assignment Problems

Problem Definition

Bounds and Linearizations

Fast Flow Formulations

Computational Experience

A Facility Lay-Out Problem

Problem Description

Problem Description

Problem Description

- The Original Lay-out and Production Flows
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0	∢					J				,	•[4		¥		
	0,1L	0,25M	0,3L	0,25M	0,5L	0,25M	4 0,7L	0,25M	0,9L	0,25M	11 L + 0,25K	0,25M		12 L + 0,75K	0,25M
Δ	1	Ď ´	2		e		4		5		1			12	
	0,1L	0,75M	7 9.3Ľ ´´	0,75M	0,5L	0,75M	9 0,7L	0,75M	10 0,9L	0,75M		L + 0.5K	0,75M		
	9	0	~		8	¢	6		10	фонтонности		13			

The Final Lay-out and Production Flows

Quadratic Assignment Problems

Problem Definition

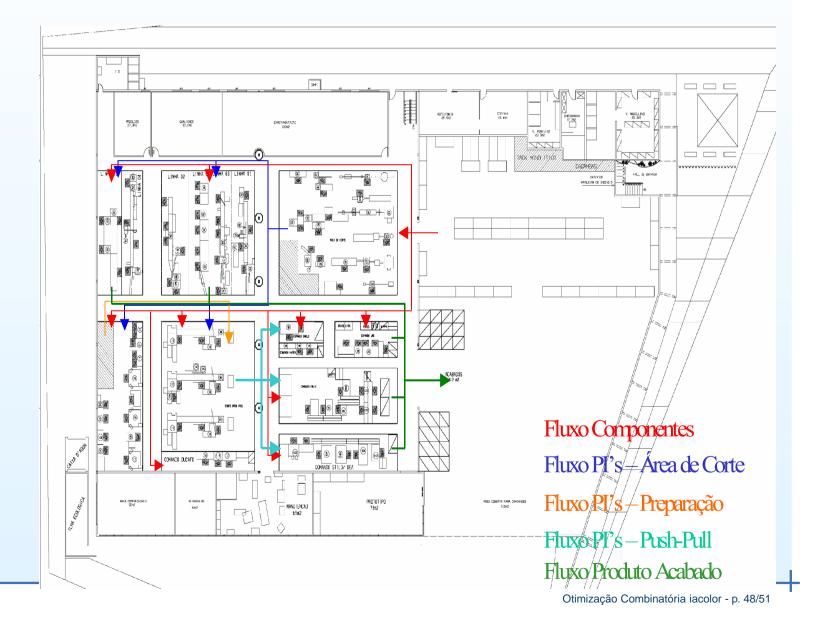
Bounds and Linearizations

Fast Flow Formulations

Computational Experience

A Facility Lay-Out Problem

- Problem Description
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Achieved Improvements

Quadratic Assignment Problems

Problem Definition

Bounds and Linearizations

Fast Flow Formulations

Computational Experience

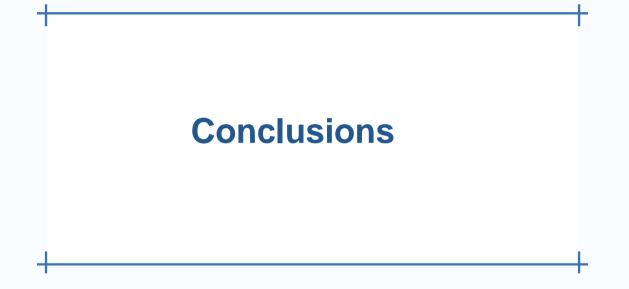
A Facility Lay-Out Problem

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Conclusions

• 12% of effective improvement of productivity.

• 15% of effective reduction of shop floor area.



Conclusions

Quadratic Assignment Problems	 Miranda and Luna Fast Flow Formulation is more sensitive to the presence of linear terms.
Problem Definition Bounds and Linearizations	 The impact of preprocessing is huge for Adams and Johnson Formulation.
Fast Flow Formulations	
Computational Experience A Facility Lay-Out Problem	 The use of a linear term appears to be a clue to break down the excessive symmetry of QAP.
Conclusions Conclusions	

• Interpreting the QAP as a network flow problem enables the solution of practical engineering problems.