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Int. J. Production Economics 58 (1999) 253–264

international journal of production economics

# A mathematical programming approach to evaluating alternative machine clusters in cellular manufacturing

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Received 22 July 1996; accepted 27 July 1998

**Abstract**

Cellular manufacturing systems achieve the economies of scope and scale approaching that of flexible and high-volume production when the machine/part clusters are totally independent of each other. However, most real systems contain bottleneck machines and exceptional parts (exceptional elements) that reduce these economies. Many grouping methods have been proposed for creating the initial machine/part cells where the presence of exceptional elements may greatly affect their performance. Furthermore, multiple alternative solutions are often possible for a given grouping algorithm. In this paper, the previous work dealing with exceptional elements is reviewed. A mathematical programming model used for comprehensively dealing with exceptional elements is investigated. The effect of alternative initial machine/part clusters on the total cost is evaluated. It is demonstrated that the mathematical programming model can provide useful information in making trade-off decisions when exceptional elements are present. 

*Keywords:* Cellular manufacturing; Exceptional elements; Mathematical programming; Group technology; Cluster analysis; Tradeoff analysis; Cost analysis

## 1. Introduction

Cellular manufacturing (CM) has received considerable attention from both practitioners and researchers for the past two decades. CM is a special application of group technology (GT) which is used to cluster parts into families and machines into cells for efficient production. By implementing CM one can take advantage of parts similarities in design attributes and manufacturing characteristics to achieve the benefits of reduced setup time, reduced manufacturing lead time, reduced material handling costs, reduced design duplication, and reduced work-in-process inventory [1,2].

Designing cells totally independent of each other is one of the major goals that enable CM systems to realize the advantages of both the mass production and the job shop environments [3]. In the ideal CM layout, all operations of parts in a family are completed within a single machine cell. However, in less than ideal situations – as is often the case in the real-world – fully independent CM clusters do not

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0925-5273/99/$ – see front matter 
PII: S0925-5273(98)00211-4

always occur; for example, in a survey of industry users of cellular manufacturing, Wemmerlov and Hyer [4] find that 20% of those with manned cells and 14% of those with unmanned cells have situations where machines were shared between cells. Furthermore, the authors report that only in about 78% of the cases do the processing times for parts occur within a single cell. Parts that are processed by more than one machine cell (i.e., exceptional parts) and machines that are required by two or more part families (i.e., bottleneck machines) are known collectively as exceptional elements (EEs). Exceptional elements are counterproductive to the philosophy of cellular manufacturing in that they

1. disrupt the philosophy of CM to create an independent environment,
2. reduce the benefits (e.g., economies of scale, and setup time and cost reduction) of CM, and
3. cause the cost of intercellular movements.

In academic research, a few authors, e.g., Kern and Wei [5], Shafer et al. [6], Logendran [7,8], Wei and Gather [9], Seifoddini [10], Offodile [11], and King and Nakornchai [12] recognize the problem caused by EEs. However, of these authors, only Kern and Wei, Shafer et al., Logendran, and Wei and Gaither address the problem enough to propose some solution. Perhaps, this is due to the non-trivial nature of the problem. Indeed, the classical 0–1 machine-clustering problem is a traveling salesman problem [13], which belongs to a class of problems classified in the literature as NP-complete. Consequently, examples are often constructed for CM grouping algorithms that do not contain exceptional elements. Usually this is done for simplicity and illustrative purposes but the grouping method may not produce useful solutions in practice.

Choosing an appropriate cell formation method is an important decision in designing a CM system. As pointed out by Shafer and Meredith [14] and Mosier [15], different cell formation methods often result in a different number of cells or solutions, and a different number of EEs if they exist. Typically, a manufacturing cell consists of functionally dissimilar machines that are used for processing of part families. Thus, it should be expected that owing to the dissimilarity of the machines, attainment of mutually separable clusters of machine cells would not be possible in all situations. Several measures of performance such as number of EEs and intercellular movements [16–18], bond energy and bond strength [19,20], grouping efficiency [21,22], and grouping efficacy [23] have been proposed for evaluating the effectiveness of different cell formation methods. A comparison of nine grouping methods on 68 problem sets using three different grouping measures found that none performed best in every situation [24]. The advantage of the mathematical programming method is that it is optimizing and should result in the same solution so long as their prevailing assumptions are not violated.

Even beyond such measures, other practical considerations such as immovable machines will in some manner affect the groupings to be considered. Wemmerlov and Hyer [25] classify these considerations into structural and operational issues. Structural issues include the selection of tools, fixtures, material handling equipment, and the choice of equipment layout. Operational issues include formulation of maintenance policies, scheduling, inspection policies, reporting mechanism, and job responsibilities. Grouping algorithms vary in their ability to account for these issues and influence the desirability of the resultant clusters. The large number of different algorithms proposed in the literature attest to the significance and difficulty in solving the GT problem and in dealing with these issues.

Unfortunately, much of this CM research centers on the development of manufacturing cell design rather than on the merits of the design for a given manufacturing situation. Most firms will face a trade-off between operational and strategic costs that are largely ignored [26]. From a practical perspective, directly evaluating the alternative machine clusters themselves is a more salient issue than evaluating the formation method used to develop them. A primary concern in this evaluation is the presence of exceptional elements.

Exceptional elements cause additional costs in operating a CM system and hence they should be dealt with in a cost-effective manner. Although several approaches have been proposed in the literature, most of them are limited in scope or are not cost optimal. Shafer et al. [6] present a mathematical programming model to find the optimal costs related to EEs for a given part family/machine cell matrix. Three popular, pure strategies to deal with EEs – machine duplication, part subcontracting, and intercellular movement – are considered. The optimal solution can be a mixture of the three. In our view, the model of Shafer et al. [6] is sound and provides a systematic method by which decision makers can effectively choose among the different options for dealing with EEs.

A basic assumption of the mathematical programming model, as previously presented, is that an initial cell formation matrix has been developed by a grouping method. However, as noted, different grouping techniques typically result in different starting matrices while alternative solutions are often possible for a given grouping method. In this paper, we investigate the effect of the alternative starting part family/machine cell clusters on the solution of the mathematical programming model. We show that the mathematical programming model is an appropriate vehicle for analysis of the significant differences that alternative starting matrices can have on the total costs. Furthermore, we provide insights into how trade-off decisions can be made by CM designers based on the individual cost components and on the sensitivity analysis of the mathematical programming model solution. The mathematical programming model, therefore, can be used to address the operational and strategic trade-off issues important in GT implementation but largely ignored in the literature [26].

The remainder of this paper is organized as follows. First, previous works addressing the EE problem are reviewed. Then the mathematical programming model is presented and illustrated by numerical examples. Alternative starting clusters that were formed following common cell formation procedures with consideration of structural and operational issues are used for illustration. Finally, a discussion of the implications of the results and directions for future research are presented.

## 2. Related work

While the published literature concerning CM grouping algorithm numbers in the hundreds [3], we have found papers addressing the EE problem to be much less common. A few papers focus exclusively on the effects of the different EE elimination methods on system performance or cost. Often, though, the concern with EEs is addressed as part of the grouping method being proposed.

EEs can be eliminated through several methods. The duplication of bottleneck machines for the elimination of intercellular moves dates back at least to McAuley [27]. Burbidge [28] discusses five approaches for removing EEs: (1) the part can be redesigned, (2) the manufacturing process can be changed, (3) the exceptional part can be rerouted within the cell, (4) the cells can be modified to accommodate the EEs, or (5) the exceptional part can be subcontracted. Chan and Milner [17] also discuss methods for elimination of EEs. As Burbidge [28] does, they consider redesigning and rerouting exceptional parts. Additionally, they introduce the concept of remainder cell which is an independent cluster containing all the EEs. Furthermore, they discuss the duplication of bottleneck machines as well as simply allowing the intercellular transfers to remain.

King [29] first proposes a rank order clustering (ROC) algorithm in which binary weights are assigned to each row and column of the original machine-part matrix. These weights are then converted to their decimal equivalence and rearranged in their descending order of magnitude. The algorithm converges in two iterations and if mutually separable clusters exist, they will be found. Otherwise, the bottleneck machines are duplicated to conform to the desired block diagonal pattern after the sorting of the machine-part matrix. However, the ROC solution is sensitive to the initial ordering of the machine-part incidence matrix. The solution can therefore result in partially separable clusters for a problem that has mutually separable clusters [18]. Because of the weighting process, there is the potential for a large increase in problem dimension, which taxes the computer memory, with a corresponding resultant cost. King and Nakornchai [12] address this concern by employing a much quicker sorting procedure (called ROC2) in which the EEs that result after the first two iterations of the ROC are ignored. The ROC is then repeated on the truncated problem to reveal a block diagonal pattern. Depending on the resulting number of blocks, they could be merged and the EEs duplicated to conform to the number of block diagonals.

A two-stage heuristic approach for designing a CM system is given by Choobineh [30]. Stage one consists of using a clustering technique with a proximity measure to form part families. The second stage utilizes a cost-based integer programming model for machine cell formation, which allows for machine duplication. Sule [31] and Sarker and Yu [32] also develop two-stage procedures for finding the optimal levels of machine duplication. These procedures compare the costs of machine duplication to those for allowing intercellular movement to remain. Shafer and Rogers [33] present a goal-programming approach to cell formation in which EEs can be handled by equipment purchases or allowing the intercellular movements to remain. Considerations of setup times and machine utilization are simultaneously achieved. A two-stage heuristic procedure is also presented to simplify the solution process.

Askin and Chiu [34] present a heuristic grouping procedure that allows for machine duplication and intercellular moves. Logendran [8] proposes a conceptual model for determining appropriate bottleneck machine duplication where machine duplication costs and intercellular movement costs are considered. Gunasingh and Lashkari [35] employ an integer programming approach for allocating machines to part families, which are assumed known. Their formulation is cost-based and allows for both machine duplication and the possibility of having some intercellular movements remain.

Seifoddini and Wolfe [36] introduce a machine duplication procedure based on the number of intercellular moves resulting from each bottleneck machine. Machines with the highest number of resultant intercellular moves are duplicated first and the procedure is repeated until an intercellular move threshold value is reached. Alternative solutions can be investigated by changing the threshold value. Seifoddini [10] formulates a cost-based machine duplication process that compares the machine duplication costs to the savings from reduced intercellular movement.

In all the papers discussed above, elimination of EEs is mainly limited to duplication of bottleneck machines. While many do allow intercellular moves to remain, all ignore the possibility of subcontracting exceptional parts. Some of the methods are cost based but most are not. Also, note that the heuristic procedures, which generally find good solutions, are not guaranteed to be optimal.

A different approach is to eliminate EEs by subcontracting the exceptional parts. Kumar and Vannelli [37] present two algorithms to identify the minimal number or the minimal total cost of subcontracting exceptional parts. Vannelli and Kumar [38] extend their work by allowing subcontracting, machine duplication, or both. Wei and Gather [9] give an optimal 0–1 integer programming model that minimizes the intercellular movement and subcontracting costs related to exceptional parts subject to machine capacity constraints. Kern and Wei [5] suggest a procedure using machine duplication and subcontracting for EE elimination. In their model, however, a mixed strategy of both duplication and subcontracting is not possible in elimination of a specific EE, making the model less flexible.

Shafer et al. [6] propose a mathematical programming approach to dealing with EEs that allows for the possibility of a mixture of strategies for specific EEs. This is an important advance over traditional pure strategies in eliminating EEs because of its flexibility to determine the cost optimal configuration based on as many of the cost components as justified. This capability can result in considerable savings compared to other traditional methods.

## 3. Mathematical programming model

The mathematical programming model for comprehensively dealing with EEs, first introduced by Shafer et al. [6], explicitly considers three cost categories: (1) subcontracting costs; (2) machine duplication costs; and (3) intercellular transfer costs. Under the assumption that an initial cell formation solution, and hence the EEs, have been found, the basic model is given as follows:

$$\text{Min} \quad \sum_{f} \left\{ \sum_{i \in G_f} X_i S_i + \sum_{k \in H_f} Y_{kf} A_k + \sum_{i \in G_f} Z_{ik} I_i \right\} \tag{1}$$

$$\text{s.t.} \quad Z_{ik} = D_i - X_i - (C_k M_{ik} / P_{ik}) \quad \forall \text{ EEs}, \tag{2}$$

$$\sum_{i \in G_f} M_{ik} \leqslant Y_{kf} \quad \forall\, k,f, \tag{3}$$

$$X_i,\, Y_{kf},\, Z_{ik} \quad \text{all integer}, \tag{4}$$

where the decision variables are

$X_i$ = units of part $i$ to be subcontracted,
$Y_{kf}$ = number of machines of type $k$ to be purchased for cell $f$,
$Z_{ik}$ = number of intercellular transfers required by part $i$ because of no machine type $k$ available within the part's cell,
$M_{ik}$ = number of machines of type $k$ dedicated to production of part $i$ (utilization of machine type $k$ to produce part $i$)

and the major parameters are

$A_k$ = annual total cost of a machine of type $k$,
$S_i$ = incremental cost of subcontracting a unit of part $i$,
$I_i$ = incremental cost for moving part $i$ outside of a cell,
$C_k$ = annual capacity of machine type $k$,
$D_i$ = annual demand for part $i$,
$P_{ik}$ = processing time of part $i$ on a machine of type $k$,
$G_f$ = set of exceptional parts in cell $f$,
$H_f$ = set of bottleneck machines required by parts in cell $f$.

The above model is a mixed integer program (MIP). In the objective function (1), the first component represents the subcontracting cost, the second represents the machine duplication cost, and the third represents the intercellular transfer costs associated with remaining EEs. Constraint (2) is a logical balance on the number of intercellular transfers for exceptional elements. Constraint (3) ensures that the total number of machines of type $k$, used for production of exceptional parts in a cell, does not exceed the number of type $k$ machines to be purchased for that cell. Constraint (4) represents the necessary integer restrictions for variables $X_i$, $Y_{kf}$, $Z_{ik}$.

This model explicitly considers total cost as the basis in choosing the best combination of the strategies for designing a CM system. It has practical value and is very flexible in real-world situations for dealing with EEs. The parameters employed in the model are generally available for practical situations. The cost function and the constraints can be expanded to meet the varied, individualized requirements of different CM environments. It also provides a systematic approach for management to evaluate the economic consequence of EEs from both operational and strategic perspectives.

In their original formulation and solution, Shafer et al. [6] relax the integer restriction on the variables $X_i$ and $Z_{ik}$ for simplicity and solve it using LINDO [39]. In our study, we avoid this relaxation step by using the IBM's Optimization Subroutine Library (OSL) [40] on an RS/6000 Model 530 workstation. The OSL contains many user controls and callable routines for solving large-scale linear programs, mixed integer programs, and quadratic programs [41]. Without the relaxation, the problems are more complete, realistic, and the solution more practical. Thus, OSL makes the solution of MIP problems of the magnitude likely to be encountered in cellular manufacturing possible and more efficient.

One of the major assumptions in using this model is that the machine/part grouping is completed first. However, as discussed earlier, initial machine/part matrices can have an effect on the total cost of the EEs. In the next section, six different starting machine/part clusters are formed and used to evaluate the model. These machine/part clusters are found using the single linkage clustering (SLC, [24]) and ROC [29] methods. These methods are chosen because of their popularity and because they are known to sometimes result in alternative solutions. For example, SLC can yield different solutions when alternative threshold levels are used or when equal similarity coefficient values occur. The ROC solution, on the other hand, can be sensitive to the initial ordering of the machine-part incidence matrix [18].

Once these machine/part clusters have been found, we employ the mathematical programming model and the parameters presented in Shafer et al.'s [6] example to compare their economic performance due to the EEs. Our objective here is not necessarily to evaluate and compare different cell formation methods. Some authors [14,24] have done this. Rather, our major purpose is to see how the initial formations of machine/part clusters can influence the total costs relating EEs, how these costs could easily be determined and analyzed using a mathematical optimization model, and to illustrate guidelines for practical use of the model.

## 4. Evaluating alternative cluster formulations

Fig. 1 contains the costs, capacities, and processing information needed to run the mathematical model. The entries in the main section of this figure represent processing times while the other entries represent costs, demands, and capacities. For example, part 1 will be processed on machines 1–4 and 6. The processing times will be 2.95, 2.76, 5.54, 2.91, and 1.92 minutes respectively. Fig. 2 gives the six alternative cell-clustering results. The four cluster alternatives (Fig. 2a–d) are formed using SLC while the two cluster alternatives are based on ROC (Fig. 2e–f). The desirability of one cluster arrangement compared to another is not an issue here, but rather the analysis of their effect on the cost optimization process using the mathematical programming model is.

Using the MIP formulation Eqs. (1)–(4), for the above six different starting cell formations we obtain the results summarized in Table 1. It is obvious that employing the mathematical programming model may produce a mixed strategy to deal with exceptional elements and that the optimal mixed policy may have substantial benefits compared to a traditional, pure strategy for dealing with EEs. As can be seen from Table 1, the total cost for each of the mathematical programming methods is significantly lower than those from each of the pure strategies. The MIP solution of Alternative 1, for example, has the minimum cost at $460,183.60. The cost of using machine duplication alone to eliminate all intercellular moves will be $641,937. The costs of pure intercellular moves and subcontracting all exceptional parts will be $652,862 and $782,262, respectively. We can see that the combination of all three EE elimination strategies can significantly reduce the total cost.

It should be noted here that the actual cost of the original cell formation solution is that of the pure intercellular move strategy. As shown in Table 1 and explained already, these costs can be significantly reduced by employing the mathematical programming model. Notice that the combined MP strategies are superior to their corresponding pure intercellular move strategies. In the case of Alternative 1, the improvement in the MP mixed strategy over its corresponding pure strategy is about 42%.

Table 1 also shows that as a commonly used clustering effectiveness measure, the number of EEs may not be appropriate from a cost perspective. Note that Alternative 2 has the smallest number of EEs for the three-cell alternatives but it has the second highest cost among them. Therefore, a company deciding to implement Alternative 2 on the

PARTS

| MACHINES | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | A(k) | C(k) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2.95 | | 2.20 | | | | | | | 4.61 | 50,784 | 2,000 |
| 2 | 2.76 | 5.18 | 1.89 | 3.89 | | 5.14 | | | | | 67,053 | 2,000 |
| 3 | 5.54 | 4.29 | | | | | | | | | 43,944 | 2,000 |
| 4 | 2.91 | | | 1.97 | 2.59 | 4.01 | | 2.70 | | | 67,345 | 2,000 |
| 5 | | | | 4.28 | | 4.51 | | | | | 42,414 | 2,000 |
| 6 | 1.92 | | | | | | 2.23 | | 5.52 | | 75,225 | 2,000 |
| 7 | | | | | 3.40 | | 1.16 | 4.72 | | 2.49 | 52,741 | 2,000 |
| 8 | | 5.32 | | | | | | 3.75 | 3.85 | | 63,523 | 2,000 |
| 9 | | | | | | | 4.04 | | | 1.83 | 50,632 | 2,000 |
| S(i) | 4.2 | 4.3 | 3.5 | 4.4 | 5 | 3.9 | 4.4 | 4.6 | 5 | 5 | | |
| D(i) | 32,128 | 27,598 | 20,651 | 11,340 | 18,707 | 17,040 | 46,196 | 45,384 | 16,409 | 22,000 | | |
| I(i) | 3.7 | 2.8 | 2.8 | 3.3 | 2.8 | 3.5 | 2.8 | 2.6 | 3.4 | 3.2 | | |

Fig. 1. Demand, capacity, cost, and processing information.

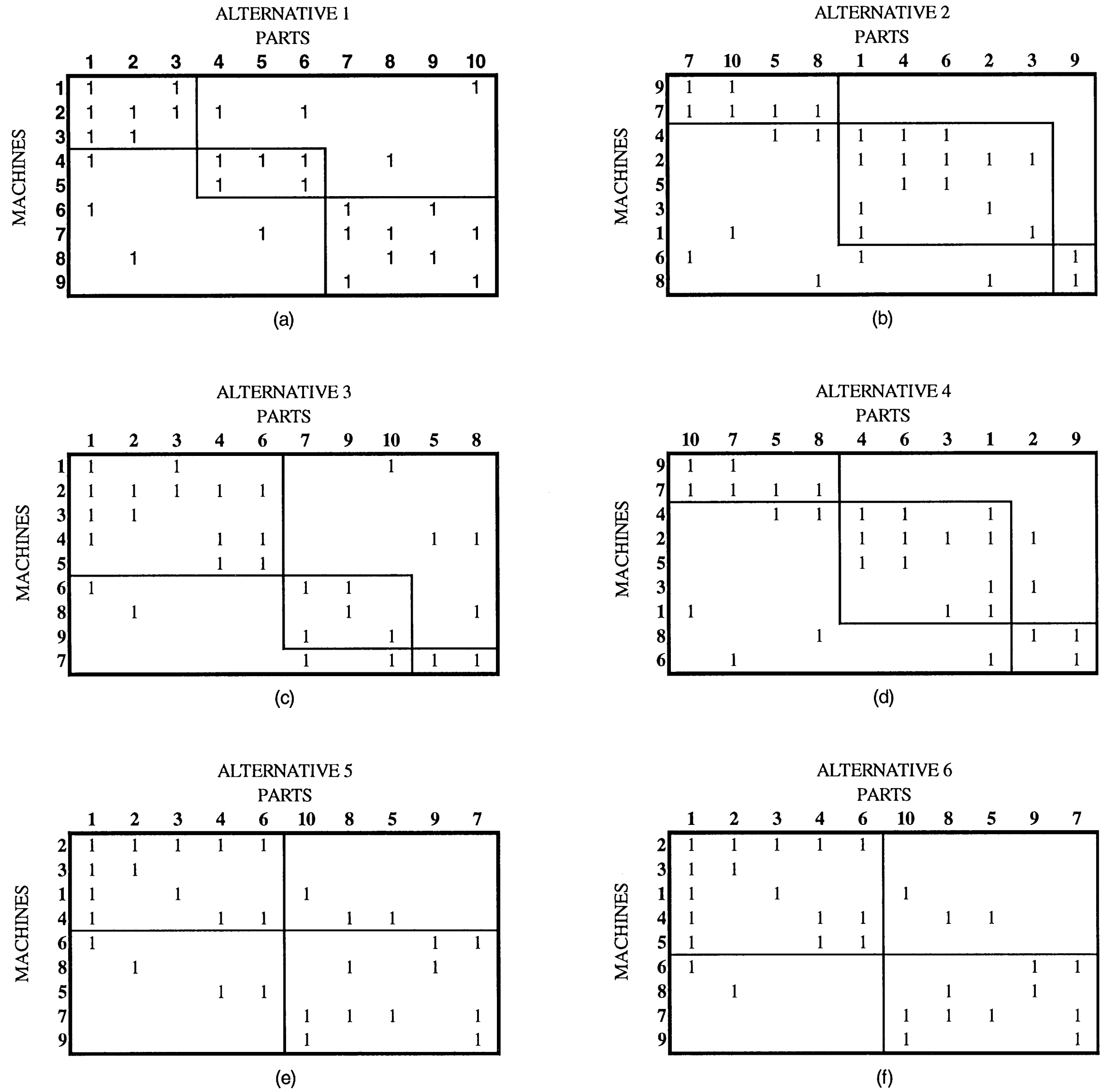


Fig. 2. Alternative cluster formations.

basis of the number of EEs would pay a 5% total cost premium over Alternative 1.

The results for the three cluster alternatives also illustrate the cost differences for configurations with the same number of EEs. In Table 1, three of the groupings contain eight EEs. However, a range of \$64 527 or 14% exists between the highest cost configuration (Alternative 4) and the lowest (Alternative 1). Certainly, management would consider cost differences of this magnitude in deciding the configuration best suited to its needs. Given the problem parameters, the mathematical programming approach provides this important cost information enabling management to evaluate possible options. At the same time, it insures that each alternative configuration is cost optimal.

Table 1
Mathematical program (MP) and pure strategy summary results

| Alt. | No. of cells | No. of EEs | MP total cost | MP cost components | | | Pure strategy cost | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | Machine duplication | Part subcontracting | Intercellular moves | Machine duplication | Part subcontracting | Intercellular moves |
| 1 | 3 | 8 | $460 183.60 | $185 182.00 | $134 937.60 | $140 064.00 | $641 937.00 | $782 262.00 | $652 861.60 |
| 2 | 3 | 7 | $483 313.60 | $332 102.00 | $61 566.40 | $89 645.20 | $590 016.00 | $869 172.80 | $684 273.20 |
| 3 | 3 | 8 | $462 761.40 | $309 618.00 | $61 566.40 | $91 577.00 | $567 531.00 | $869 172.80 | $754 673.20 |
| 4 | 3 | 8 | $524 710.60 | $332 102.00 | $180 237.00 | $12 370.80 | $641 020.00 | $869 172.80 | $761 547.60 |
| 5 | 2 | 7 | $364 364.20 | $235 768.00 | – | $128 596.20 | $472 573.00 | $782 262.40 | $533 988.00 |
| 6 | 2 | 5 | $317 797.60 | $193 354.00 | – | $124 443.60 | $387 745.00 | $665 910.40 | $436 926.00 |

Determining the optimal number of cells is a controversial issue in CM. This decision is highly subjective and often is based on personal judgment with consideration of available spaces, production volume, setup times, the effect of cell interdependence, etc. [42]. Knowledge of the cost impact in making the operational and strategic tradeoff decisions inherent in choosing among the different alternatives would be valuable.

Alternatives 5 and 6 in Table 1 contain the results for two cluster groupings. It can be seen that the cost optimal solution for two clusters is over $142 000 less than the minimum cost for the three cluster alternatives – a decrease of nearly one-third. It is obvious that as the number of cells approach the number of machines, the cost of inter-cellular movements will increase. Fewer parts are processed per cell, thereby increasing the likelihood of intercellular movement costs. Thus, loss of cell autonomy could be compared with this cost difference to determine if it is justified.

In addition, note that neither of the two cluster alternatives results in subcontracting of parts. Furthermore, the two cluster options do not require purchasing machines in any greater numbers than the three cluster alternatives. Therefore, if the exceptional parts are strategically important for the organization to produce or if the firm is averse to additional capital investment, the two cluster alternatives may be attractive.

Now, to gain more insight into the problem structure and solution, we consider one of the alternative cluster formations in detail. Alternative 3 in Fig. 2c is chosen to illustrate the use of all three pure strategies for the elimination of a single EE. Table 2 contains specific information for the solution of Alternative 3 while Table 3 contains OSL output for Alternative 3.

Alternative 3 is a three-cluster example with 8 EEs, some of which have a greater impact on the optimal solution. For example, consider exceptional part 8, which has a relatively high demand and requires processing on machines in all three cells (see Fig. 2c). Compare it to exceptional part 1, which has a lower demand and is associated with only two of the cells. Note that the processing times of part 8 is higher than that of part 1 as far as the bottleneck machines are concerned. However, the intercellular movement cost of part 8 is lower than that of part 1 while its subcontracting cost is higher. These relative characteristics of exceptional part 8 suggest that the optimal solution may involve a mixed strategy.

Indeed, Table 2 shows that to deal with exceptional part 8, all three strategies for dealing with the EEs should be used. Under a pure intercellular move policy, part 8 has 90,768 intercellular moves because the 45 384 parts have to be processed in the other two cells. In the cost optimal solution, only 5501 intercellular moves remain with the rest being eliminated via machine duplication and subcontracting. The OSL output in Table 3 shows that both machines 4 and 8 are purchased for cell three while machines 1 and 7 are duplicated for cell two

Table 2
Alternative 3 solution detail

| Exceptional parts | Initial intercellular move | Units to subcontract | Intercellular moves eliminated due to new machine | Remaining intercellular moves |
|---|---|---|---|---|
| 1 | 32 128 | 0 | 32 128 | 0 |
| 2 | 27 598 | 0 | 0 | 27 598 |
| 7 | 46 196 | 0 | 46 196 | 0 |
| 10 | 44 000 | 0 | 44 000 | 0 |
| 5 | 18 707 | 0 | 18 707 | 0 |
| 8 | 90 768 | 13 384 | 58 499 | 5501 |

Total cost = \$462,761.4

Table 3
OSL output for Alternative 3

| Variable | Activity | Dual | Lower limit | Upper limit |
|---|---|---|---|---|
| X1 | | 4.2 | | Infinity |
| X2 | | 1.5 | | Infinity |
| X7 | | 4.4 | | Infinity |
| X10 | | 5.0 | | Infinity |
| X5 | | 5.0 | | Infinity |
| X8 | 13 384 | 2.0 | 13 384 | Infinity |
| Y1,2 | 1 | 50 784 | 1 | Infinity |
| Y4,3 | 1 | 67 345 | 1 | 1 |
| Y6,1 | 1 | 75 225 | 1 | 1 |
| Y8,1 | | 388.852 | | Infinity |
| Y8,3 | 1 | − 19 677 | | 1 |
| Y7,2 | 1 | 52 741 | 1 | Infinity |
| Z1,6 | | 3.7 | | Infinity |
| Z2,8 | 27 598 | | | Infinity |
| Z7,7 | | 2.8 | | Infinity |
| Z10,1 | | 3.2 | | Infinity |
| Z10,7 | | 3.2 | | Infinity |
| Z5,4 | | 2.8 | | Infinity |
| Z8,4 | 5501 | 2.6 | 5501 | Infinity |
| Z8,8 | | | | Infinity |
| M1,6 | 51.40% | | | Infinity |
| M2,8 | | | | Infinity |
| M7,7 | 44.66% | | | Infinity |
| M10,1 | 84.52% | | | Infinity |
| M10,7 | 45.65% | | | Infinity |
| M5,4 | 40.38% | | | Infinity |
| M8,4 | 59.62% | | | Infinity |
| M8,8 | 100.00% | | | Infinity |

and machine 6 for cell one. The machine clusters in the optimal solution to Alternative 3, therefore, are comprised of machines 1–6 in cell one; machines 1 and 6–9 in cell two; and machines 4, 7, and 8 in cell three. Intercellular moves, meanwhile, are eliminated for every exceptional part except parts 2 and 8.

The OSL output in Table 3 contains information from the optimal mathematical programming solution that can be used by designers for sensitivity analysis. The first column in Table 3 gives the variable name. The second column contains the variable's value in the optimal solution with blanks representing a level of zero. The dual column shows how much the objective function's coefficient for that variable would have to change before the basic solution changes. The lower and upper limit columns specify the range of values of the variable's activity for which the current basic variables remain the same. This sensitivity information provides valuable information concerning areas for targeting process improvement activities.

The dual prices for the variables in Table 3 contain information concerning which parts are potentially attractive (or unattractive) for subcontracting. Notice that the dual variables associated with the $X_i$ variables (subcontracting levels) indicate that for exceptional parts 1, 5, 7, and 10 the current basic solution remains valid until the cost of subcontracting drops to zero. Therefore, these parts can effectively be excluded from subcontracting considerations. However, parts 2 and 8 have dual prices of \$1.5 and \$2.0, respectively. These prices suggest that if the subcontracting costs for parts 2 and 8 could be decreased to the same rate as their intercellular move costs, the current basis would change and resolving the model would result in a different optimal mixed strategy. The firm may therefore wish to work with current vendors to make the supply chain more efficient or seek bids from new suppliers of these parts.

The dual prices for the $Y_{kf}$ variables indicate that a basis change does not occur until the entire machine cost coefficients drop to zero except machine 8 for cells 1 and 3. The machine duplication annual cost would have to change by only \$389 per year for machine 8, cell 1 for the basic solution to change. This represents only six-tenths of one percent of the annual machine costs for machine 8 and management may therefore wish to review the maintenance and operational procedures to see if this small saving can be realized. Finally, the dual variables for $Z_{ik}$ show that the current basis is optimal until the cost of intercellular moves drops to zero.

## 5. Direction for future research

The basic model given in Eqs. (1)–(4) can be expanded to reflect production limitations or managerial policies. For example, the following constraints:

$$\sum_{i \in G_f} X_i S_i \leqslant C_x, \tag{5}$$

$$\sum_{k \in H_f} \sum_{f} Y_{kf} A_k \leqslant C_y, \tag{6}$$

$$\sum_{i \in G_f} \sum_{k} Z_{ik} I_i \leqslant C_z, \tag{7}$$

may be added to the basic model. They represent capacity restrictions on subcontracting, machine duplication, and intercellular transfer costs, respectively. If machine duplication costs need to be kept below a certain budgetary limit, $C_y$, Eq. (6) should be used. Setting any two of the right-hand sides of Eqs. (5)–(7) equal to zero, and the third to a large value, will force the pure strategy solution as presented in Table 1. The dual values to these constraints represent the amount that the objective function would improve given a unit change in the right-hand side. This information is useful for management to determine the effect the capacity constraints have on the total cost and to act accordingly.

Although the model presented in this research addresses the major issues in EEs, other issues such as intra-cellular movement, relocating production machines and multiple objective criteria should be investigated. Thus, Eq. (1) could be expanded to include the cost of intra-cellular movements and relocation of production machines. In other words, the processing and sequencing requirements of the parts could be included in the model for a truly efficient production. Heretofore, it has been implicitly assumed that the within cell scheduling and processing requirements of the parts are insignificant. Also, a more robust objective function based on goal programming and other technologies could be used to incorporate the objectives of minimizing setup and production costs and, maximizing machine utilization.

Offodile et al. [3] notes, the literature is replete with algorithms for solving the well-structured machine-part 0–1 incidence matrix with only very few addressing problems of the machine loading variety. The well-structured problems result in mutually separable clusters. However, in the real world, EEs occur and the problem becomes how to best manage them. Shafer and Meredith [14] compared selected machine-part formation methods and find that some of them work best under certain conditions. Another possible extension of this research would be to conduct a more comprehensive evaluation of the numerous machine-part clustering algorithms with the objective of developing taxonomy of their capabilities and limitations. Then, a select number of problems can be solved with the most appropriate methods and compared with the mathematical model as illustrated in this research.

## 6. Summary

Cellular manufacturing is gaining popularity because of the potential benefits that can be realized in many manufacturing systems. However, there are various problems in the design of a CM system. One critical issue is how to classify parts into families and machines into cells to create an independent environment for efficient, flexible production. Group technology is essential in the formation of machine cells/part families in designing a CM system. Although considerable amount of research has been published on the methods for grouping part families or machine cells, few of them focus on the practical or economic consequences of exceptional elements, which often occur in real manufacturing systems. The existence of EEs not only violates the independence assumption of machine-cells or part-families but also causes implementation problems. Traditional actions taken to remove EEs include machine duplication, part subcontracting, and even allowing intercellular movements to remain. However, it is often difficult to evaluate which one is the best from a cost perspective since there are other practical constraints involved in the decision process.

The mathematical programming model originally proposed by Shafer et al. [6] is a comprehensive tool to alleviate the possible economic effect due to the presence of EEs. Their model is flexible and often leads to a mixed policy, which is the most cost effective compared to traditional pure strategy in dealing with EEs. However, there is a limitation in their model assumption, that is, the model cannot be used until an initial cell formation solution is specified. An earlier study, by Shafer and Meredith [14] shows that the part family grouping procedures performed relatively well compared to the machine-part grouping procedures. We showed that different starting clusters, which can result from clustering methods like SLC or ROC, for example, could have significant economic consequences on the total cost. It is also illustrated that the mathematical programming model is a more appropriate vehicle than EEs to analyze these cost differences especially considering the operational and strategic trade-offs necessary in practice. Consequently, the model provides valuable information needed by practitioners to make decisions in many important CM design aspects.

## Acknowledgement

The authors thank the regional editor and the reviewers for their suggestions, which have helped to improve the paper.

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