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Optimal control of a single link manipulator including motor dynamics

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The nonlinear behavior of robotic manipulators motivated the research in implementing adaptive control strategies. Recently, the effects of motor dynamics have attracted the attention of a number of researchers. In this paper, we present a new optimal controller designed for rigid-body robots including motor dynamics. The new optimal adaptive controller developed in this work is based on feedback linearization, it does not require acceleration feedback, it does not assume full state is available for measurement but it requires an observer. Naturally, it does not assume exact knowledge of either robot or actuator parameters. The optimality is based on the minimization of a performance index which turns out to be possible if we could find a solution to the Hamilton–Jacobi equation.

Keywords: motor dynamics, robotics, nonlinear

Nomenclature

$(f(x), g(x), h(x))$	smooth functions where $x \in R^n$
$L_f h$	Lie derivative of $h(x)$ w.r.t f
$L_g h$	Lie derivative of $h(x)$ w.r.t g
y	system output
u	control input
v	new control input
Z	normal form transformation
θ_i	system unknown parameters for the $f_i(x)$ known function
θ_i	system unknown parameters for the $g_i(x)$ known function
$\hat{\theta}_i$	estimate of θ_i
$O(\hat{x}(t))$	observability matrix
K_{obs}	observer gain
$e = y - y_m$	output tracking error
ε	observer error
$\eta(t)$	closed loop error
V	Lyapunov function candidate
λ_m	maximum eigen value of $M(s)$
γ_M	maximum eigen value of $\xi(s)$
PI	performance index
S	smooth, positive function
u_{opt}	optimal control input
$\theta(t)$	link angle
$\omega(t)$	angular velocity of the link
J	sum of both link and motor rotational inertia
M	link mass
b	damping coefficient of the motor
R_a	armature resistance
L_a	armature inductance
i_a	armature current
K	electromechanical coefficient
l	distance from the joint axis to link center of mass

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Introduction

Control of rigid-body robot manipulators is a well studied subject for which there have been numerous contributions in the literature. An overwhelming majority of the available controllers are based on the assumption that the actuator dynamics are negligible[1–5]. This assumption reduces the dynamic model of the robot and facilitates the design of controllers. As a result of this simplification, unmodelled disturbances exist in the robot control system which affect the tracking and the positioning of the robot. Although there are several methods to make a controller robust with respect to the unmodelled dynamics, the performance of the controller is not as expected, meaning that the tracking errors are bounded but do not converge to zero[6].

The negligence of the motor dynamics is a trade-off between model accuracy and controller simplicity. The motivation and evaluation for such a trade-off is in a sense technology dependent. In the robotics early days, a linearized model was deemed acceptable in the design of controllers and performance in a specific bandwidth[2]. As a further step, a nonlinear rigid-body dynamic model has been adopted[1,3]. The rapid development of computer engineering allowed the use of a linearized model in the dynamic parameters to be used to estimate the robot inertia parameters and the design of model-based adaptive controllers[4,5]. Recently, actuator dynamics, joint flexibility and link elasticity have been included in the robot dynamic models[7].

Since many industrial robots are driven by electric motors, the motor dynamics are the major source for unmodelled high frequency disturbance. The production demands of achieving shorter cycle times demand the robot to move faster and thus the high frequency disturbances due to unmodelled dynamics can no longer be neglected since they operate in the controller bandwidth[6]. The importance of including the motor dynamics in the robot dynamic model to improve the accuracy of robot tracking and positioning was studied by Goor[8]. Inclusion of the actuators in the robot dynamic model complicates the controller structure, because the system is described by third-order differential equations. In the early studies of the control of rigid robots including actuatator dynamics[9],[10], full knowledge of the actuator parameters is assumed. Ge and Postlethwaite[11] proposed an adaptive controller for robots including motor dynamics. This requires acceleration feedback to guarantee stable tracking.

Exact linearization techniques

In this section we discuss input–output linearization of single-input nonlinear systems. Applying input–output linearization techniques we obtain a linear differential relation between the output y and a new input v, this is achieved by differentiating the output repeatedly until the input u appears and then designing u to cancel the nonlinearity. This is mathematically presented for a single-input, single-output system in Ref.[12] as follows:

$$\dot{x} = f(x) + g(x)u$$
$$y = h(x) \tag{1}$$

with $x \in R^n$; (f, g, h) smooth functions. Differentiating y with respect to time, we obtain:

$$\dot{y} = L_f h + L_g h u \tag{2}$$

where $L_f u$ and $L_g h$ are the Lie derivatives of $h(x)$ w.r.t f, g, respectively. If $L_g h \neq 0 \forall x \in R^n$ then the control law of the form $\alpha(x) + \beta(x)v$, specifically

$$u = \frac{(L_f h + v)}{L_g h} \tag{3}$$

yields the linear system

$$\dot{y} = v \tag{4}$$

The normal form is represented by applying the transformation $z = \phi(x)$ to the system eqn (1) to obtain a linear system in the new coordinates represented by

$$\dot{z} = \boldsymbol{A}z + \boldsymbol{b}v$$
$$y = Cz \tag{5}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} 0 \\ 0 \\ \dots \\ \dots \\ \dots \\ 1 \end{bmatrix},$$

$$C = [1 \;\; 0 \;\; \dots \;\; 0] \text{ and} \tag{6}$$

$$\begin{bmatrix} \phi_1(\boldsymbol{x}) \\ \phi_2(\boldsymbol{x}) \\ \dots \\ \phi_r(\boldsymbol{x}) \end{bmatrix} = \begin{bmatrix} \boldsymbol{h}(\boldsymbol{x}) \\ L_f \boldsymbol{h}(\boldsymbol{x}) \\ \dots \\ L_f^{r-1} \boldsymbol{h}(\boldsymbol{x}) \end{bmatrix}$$

Adaptive control of linearizable systems

The drawback of exact feedback linearization is that in order to cancel the nonlinear terms, we should know exactly the system parameters that constitute the and nonlinear functions. This assumption is always difficult to fulfill in practical implementations. This fact is behind the motivation for adopting adaptive control techniques to achieve asymptotic cancellation.

According to[12] for SISO systems we may represent the system as follows

$$f(x) = \sum_{i=1}^{n1} \theta_i^1 f_i(x) \tag{7}$$

$$g(x) = \sum_{j=1}^{n2} \theta_j^2 g_j(x) \tag{8}$$

where and represent the system unknown parameters for the and known functions. The estimates at time have the form:

$$\hat{f}(x) = \sum_{i=1}^{n1} \hat{\theta}_i^1(t) f_i(x) \tag{9}$$

$$\hat{g}(x) = \sum_{j=1}^{n2} \hat{\theta}_j^2(t) g_j(x) \tag{10}$$

where $\hat{\theta}_i$ and $\hat{\theta}_j$ represent the estimates of the parameters θ_i and θ_j, respectively. The control law then becomes

$$u = \frac{(-\widehat{L_f h} + v)}{\widehat{L_g h}} \tag{11}$$

where $\widehat{L_g h}$, $\widehat{L_f h}$ are the estimates of $L_g h$ and $L_f h$, respectively.

In case of adaptive tracking for SISO system the control law becomes:

$$u = \frac{(-\widehat{L_f h} + \dot{y}_{\text{ref}} + k(y_{\text{ref}} - y))}{\widehat{L_g h}} \tag{12}$$

To estimate $\hat{x}$ an observer[15] was adopted. The observer is represented by

$$\dot{\hat{x}} = f(\hat{x}) + g(\hat{x})u(t) + O^{-1}(\hat{x})K_{\text{obs}}(y(t) - h(\hat{x})) \tag{13}$$

where $O(\hat{x}(t))$ is defined as

$$O(\hat{x}(t)) = \frac{\partial}{\partial t}\begin{bmatrix} h(\hat{x}(t)) \\ L_f h(\hat{x}(t)) \\ L_f^2 h(\hat{x}(t)) \\ \cdots \\ L_f^{n-1} h(\hat{x}(t)) \end{bmatrix}$$

The observer gain K_{obs} is chosen to guarantee asymptotic stability of the estimation errors which should typically be 10 times faster than the tracking errors.

Stability of closed-loop system

As we have established in the Introduction, the nonlinear observer and adaptive controller will be used in a closed loop for trajectory tracking. The idea behind using the adaptive contoller is that the system parameters are not exactly known in practice and hence exact cancellation of the nonlinear terms is not possible. This is the motivation behind our suggestion to use adaptive control. At the same time we need state feedback but since not all the states are available or feasible to measure, we suggested use on a nonlinear observer. The output of the system composed of the adaptive controller, nonlinear observer and the plant should track the output trajectory of a reference predefined model.

Substituting eqn (11) into eqn (2) we obtain:

$$\dot{y} = L_f h_{f,g} h\left[\frac{(-\widehat{L_f h} + v}{\widehat{L_g h}}\right] \tag{14}$$

which may be written as follows:

$$\dot{y} = [\theta_1^1 \ldots \theta_{n1}^1]\begin{bmatrix} L_{f_1}h \\ \cdots \\ \cdots \\ L_{f_{n1}}h \end{bmatrix} + [\theta_1^2 \ldots \theta_{n2}^2]\begin{bmatrix} L_{g_1}h \\ \cdots \\ \cdots \\ L_{g_{n2}}h \end{bmatrix} \times \left[\frac{(-\widehat{L_f h} + v)}{\widehat{L_g h}}\right] + v - v \tag{15}$$

from eqn (11) may be expressed as

$$-v = -[\hat{\theta}_1^1 \ldots \hat{\theta}_{n1}^1]\begin{bmatrix} L_{f_1}h \\ \cdots \\ \cdots \\ L_{f_{n1}}h \end{bmatrix} - [\theta_1^2 \ldots \theta_{n2}^2]\begin{bmatrix} L_{g_1}h \\ \cdots \\ \cdots \\ L_{g_{n2}}h \end{bmatrix} \times \left[\frac{(-\widehat{L_f h} + v)}{\widehat{L_g h}}\right] \tag{16}$$

When eqn (16) is substituted into eqn (15) it yields

$$\dot{y} = v + \Theta^T W \tag{17}$$

where

$$\Theta^T = (\theta - \hat{\theta})^T,\ W = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \textit{and} \begin{bmatrix} L_{f_1}h \\ \cdots \\ \cdots \\ L_{f_{n1}}h \end{bmatrix},$$

$$w_2 = \begin{bmatrix} L_{g_1}h \\ \cdots \\ \cdots \\ L_{g_{n2}}h \end{bmatrix}\left[\frac{(-\widehat{L_f h} + v)}{\widehat{L_g h}}\right].$$

An appropriate choice for a control law used for tracking is:

$$v = \dot{y}_m + \alpha(y_m - y) \tag{17}$$

which yields the following error equation, where e is defined by $e = y\text{-}y_m$

$$\dot{e} + \alpha e = \Theta^T W \tag{18}$$

The difficulty in constructing this signal comes from the fact that is not available for measurement since it depends on $L_f h$, which in turn is not available since not all the states are measurable. A nonlinear observer has been used to overcome this difficulty. The observer error equation is as follows:

$$\dot{\varepsilon} = K_{\text{obs}}\varepsilon \tag{19}$$

where ε is represented by $\varepsilon = \hat{z}(t) - z(t)$, $z(t)$ was defined earlier as $z(t) = \Phi(x(t))$.

The differential equation for the closed-loop error is

$$\dot{\eta}(t) = A\eta(t) + B(\Theta^T W) \tag{20}$$

where

$$\eta(t) = \begin{bmatrix} \varepsilon \\ e \end{bmatrix},\ A = [-K_{\text{obs}}^T - \alpha^T]^T \text{ and } B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{20}$$

A Lyapunov function candidate is

$$V = \frac{1}{2}\eta^T\eta \tag{21}$$

Hence $V = \frac{1}{2}[\varepsilon^T\varepsilon + e^T e]$ and is positive definite.

$$\dot{V} = \frac{1}{2}\dot{\varepsilon}^T\varepsilon + \frac{1}{2}\dot{e}^T e + \frac{1}{2}\varepsilon^T\dot{\varepsilon} + \frac{1}{2}e^T\dot{e} \tag{22}$$

By substituting eqns (18),(19) and (22) we obtain:

$$\dot{V} = -[K_{\text{obs}}\varepsilon^T\varepsilon] - [\alpha e^T e] + W^T\Theta M(s)\Theta^T W \tag{23}$$

If we substitute $\xi = \Theta^T W$in eqn (23) we get

$$\dot{V} = -[K_{\text{obs}}\varepsilon^T\varepsilon] - [\alpha e^T e] + \xi^T M(s)\xi \tag{24}$$

Hence

$$\dot{V} \le -K_{\text{obs}}\|\varepsilon\|^2 - \alpha\|e\|^2 + \lambda_m\|\xi\|^2 \tag{25}$$

Since

$$\xi = \frac{e}{M(s)}, \text{ then } \|\xi\| \le \left\|\frac{1}{M(s)}\right\|\|e\|, \ \|\xi\|^2 \le \gamma_M^2\|e\|^2$$
$$\dot{V} \le -K_{\text{obs}}\|\varepsilon\|^2 - \alpha\|e\|^2 + \lambda_m\gamma_M^2\|e\|^2 \tag{26}$$

bearing in mind $\eta(t) = [{}^{\varepsilon}_{e}]$, *then eqn* (26) *yields,*

$$\dot{V} \le -(K_{\text{obs}} + \alpha - \lambda_m\gamma_M^2)\|\eta\|^2 \tag{27}$$

which may be written as

$$\dot{V} \le -\Lambda\|\eta\|^2 \tag{28}$$

A sufficient condition for $\dot{V} \le 0$ is $\Lambda > 0$ or equivalently that $(K_{\text{obs}} + \alpha - \lambda_m\gamma_M^2) > 0$ which can be guaranteed through an appropriate choice of K_{obs}, α and $M(s)$.

Optimal control of linearizable systems

Another drawback for the feedback linearization is that a considerable control effort is required to achieve a fast accurate response. In this section, an optimal control law is derived and implemented for the case of unknown system parameters and the system relative degree is equal to the system order.

Table 1 Physical parameters

R_a	L_a	K	J	b	l	Mg
1	0.002	10	4	1	1	20

Table 2 Nominal system parameters

θ_1^1	θ_2^1	θ_3^1	θ_4^1	θ_5^1	θ_6^1	θ_1^2
1	0.25	2.5	5	500	5000	500

The performance index chosen to derive the optimal control law is of the form:

$$\text{PI} = \int_0^{\infty} S[x(t), u(t)] \tag{29}$$

where S is a smooth, positive function satisfying $S(0, 0) = 0$. Such a problem has a solution if minimization of the following Hamilton–Jacobi equation has a smooth solution $V(x)$.

$$\min\left[\frac{\partial}{\partial x}V(x)\,(f(x) + g(x)u) + S(x, u)\right] = 0,\ V(0) = 0 \tag{30}$$

If we choose the function $S(x, u)$ to be:

$$S = \frac{1}{2}\int_0^{\infty}\left(\phi^T(x)C^T C\phi(x) + \frac{1}{\beta^2(\phi(x))}[u - \alpha(\phi(x))]^2\right)dt \tag{31}$$

let us define:

Table 3 Parameters and states initial conditions

$x_1(0)$	$x_2(0)$	$x_3(0)$	θ_5^1	θ_6^1	θ_1^2	y_{ref}	$\dot{y}_{\text{ref}}$
0	5	5	500	5000	550	2	−1

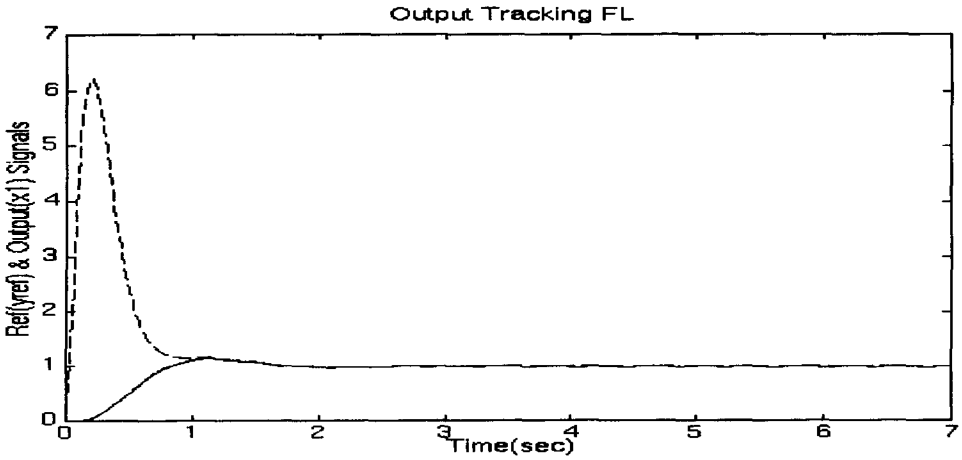

Figure 1 Tracking response in case of typical feedback linearization.

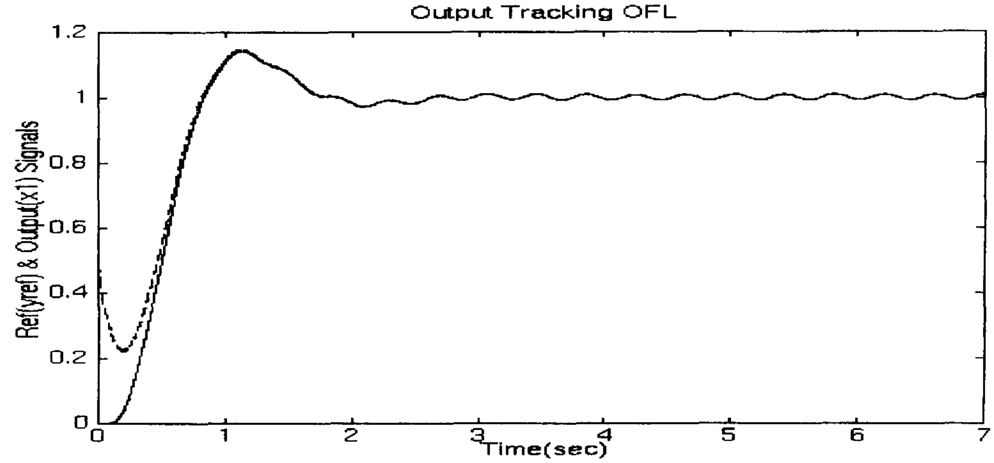

Figure 2 Tracking response in case of optimal feedback linearization.

$$\phi_e(x)=\begin{bmatrix}\phi_{1e}(\mathbf{x})\\ \phi_{2e}(\mathbf{x})\\ \dots\\ \phi_{ne}(\mathbf{x})\end{bmatrix}=\begin{bmatrix}\mathbf{h}(\mathbf{x})-\mathrm{y}_{\mathrm{ref}}\\ \mathrm{L}_f\mathbf{h}(\mathbf{x})-\mathrm{L}_f\mathbf{h}(\hat{\mathbf{x}})\\ \dots\\ \mathrm{L}_f^{n-1}\mathbf{h}(\mathbf{x})-\mathrm{L}_f^{n-1}\mathbf{h}(\hat{\mathbf{x}})\end{bmatrix}$$

$$=z-\begin{bmatrix}y_{\mathrm{ref}}\\ L_f\mathbf{h}(\hat{\mathbf{x}})\\ \dots\\ \mathrm{L}_f^{n-1}\mathbf{h}(\hat{\mathbf{x}})\end{bmatrix}=\phi(\mathrm{x})-\begin{bmatrix}\mathrm{y}_{\mathrm{ref}}\\ \mathrm{L}_f\mathbf{h}(\hat{\mathbf{x}})\\ \dots\\ \mathrm{L}_f^{n-1}\mathbf{h}(\hat{\mathbf{x}})\end{bmatrix}$$

and the observer for estimating $\hat{\mathbf{x}}$ is represented by eqn (13). The minimization of eqn (30) leads to

$$\frac{\partial}{\partial x}V(x)[g(x)]+\frac{1}{\beta^2(\phi(x))}[u-\alpha(\phi(x))]=0 \tag{32}$$

which in turn leads to:

$$u=\alpha(\phi(x))-\beta^2(\phi(x))g(x)^T\frac{\partial}{\partial x}V(x)^T \tag{33}$$

If we substitute eqn (33) into the Hamilton–Jacobi equation in eqn (30) we obtain:

$$\begin{aligned}&\frac{\partial}{\partial x}V(x)[f(x)+g(x)\alpha(\phi(x))]\\ &-\frac{1}{2}\beta^2(\phi(x))\frac{\partial}{\partial x}V(x)g(x)g(x)^T\frac{\partial}{\partial x}V(x)^T\\ &+\frac{1}{2}\phi^T(x)C^TC\phi(x)=0,\ V(0)=0\end{aligned} \tag{34}$$

One of the fundamental results from dissipative system theory as given by [13] is the following:

$$\begin{aligned}&\frac{\partial}{\partial x}V(x)f(x)+\frac{1}{2\lambda^2}\frac{\partial}{\partial x}V(x)g(x)g(x)^T\frac{\partial}{\partial x}V(x)^T\\ &+\frac{1}{2}\phi^T(x)C^TC\phi(x)=0\end{aligned} \tag{35}$$

where $\lambda>0$. By equating eqns (34) and (35) we get

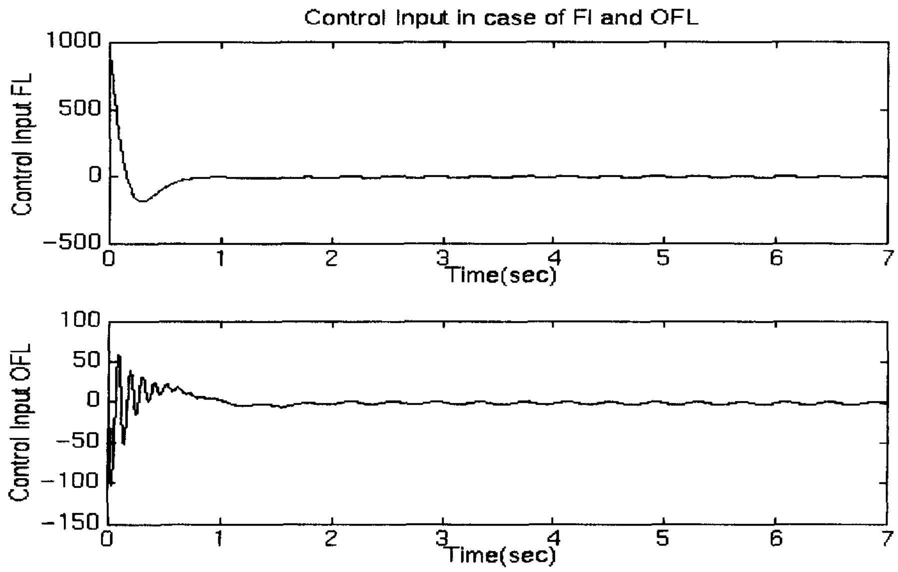

Figure 3 Control input in case of typical feedback linearization and optimal feedback linearization.

$$g(x)^T\frac{\partial}{\partial x}V(x)^T=\frac{\alpha(\phi(x))}{\left(\frac{1}{2\lambda^2}+\frac{\beta^2(\phi(x))}{2}\right)}=\frac{2\alpha(\phi(x))}{\left(\frac{1}{\lambda^2}+\beta^2(\phi(x))\right)} \tag{36}$$

If we choose $V(x)=1/2\,\phi^T(x)K\phi(x)$, where K is the unique symmetric, positive-definite solution of the algebraic Riccati equation:

$$KA+A^TK-Kbb^TK+C^TC=0 \tag{37}$$

According to Ref. 12, using the change of coordinates we can write:

$$\left[\frac{\partial}{\partial x}\phi(x)[f(x)+g(x)\alpha(\phi(x))]\right]_{x=\phi(z)},=Az$$

$$\left[\frac{\partial}{\partial x}\phi(x)g(x)\beta(\phi(x))\right]_{x=\phi(z)},=b \tag{37}$$

which in turn simplifies eqn (34) to the following form:

$$z^TKAz-\frac{1}{2}z^TKbb^TKz+\frac{1}{2}z^TC^TCz=0 \tag{38}$$

Now using eqn (37) we may substitute $\phi_e(x)$ for $\phi(x)$ in eqn (36) to obtain:

$$u_{opt}=\alpha(\phi_e(x))-\beta^2(\phi_e(x))g(x)^T\frac{\partial}{\partial x}V(x)^T \tag{39}$$

By substituting eqn (36) into eqn (39) we get

$$u_{opt}=\alpha(\phi_e(x))-\beta^2(\phi_e(x))\frac{2\alpha(\phi_e(x))}{\left(\frac{1}{\lambda^2}+\beta^2(\phi_e(x))\right)} \tag{40}$$

Case study

The case we present is of a single link manipulator arm consisting of a rigid link coupled to the shaft of an armature controlled DC motor. The model parameters are listed in *Table 1*. The state variables are chosen as follows $x_1(t)=\theta(t)$, $x_2(t)=\omega(t)$ and $x_3(t)=i_a(t)$.

The state equations are:

$$\dot{x}_1=x_2 \tag{41}$$

$$\dot{x}_2=\frac{b}{J}x_2+\frac{Km}{J}x_3+\frac{Mgl}{J}\sin(x_1) \tag{42}$$

$$\dot{x}_3=\frac{-Km}{L_a}x_2+\frac{-R_a}{L_a}x_3+\frac{1}{L_a}u \tag{43}$$

The output equation is:

$$y=h(x)=\theta(t)=x_1(t) \tag{44}$$

It is clear from the state equations that:

$$f(x)=\begin{bmatrix}x_2\\ -\frac{b}{J}x_2+\frac{Km}{J}x_3+\frac{Mgl}{J}\sin(x_1)\\ \frac{-Km}{L_a}x_2+\frac{-R_a}{L_a}x_3\end{bmatrix} \tag{45}$$

$$g(x)=\begin{bmatrix}0\\0\\ \frac{1}{L_a}\end{bmatrix} \tag{46}$$

The relative degree of this system is 1. If we define the system parameters as follows:

$$\theta_1^1=1,\ \theta_2^1=\frac{b}{J},\ \theta_3^1=\frac{Km}{J},\ \theta_4^1=\frac{Mgl}{J},\ \theta_5^1=\frac{R_a}{L_a},$$
$$\theta_6^1=\frac{Km}{L_a},\ \theta_1^2=\frac{1}{L_a}$$

The estimated system equations may be written in the following form:

$$\hat{f}_1=\begin{bmatrix}\hat{x}_2\\0\\0\end{bmatrix},\ \hat{f}_2=\begin{bmatrix}0\\-\hat{x}_2\\0\end{bmatrix},\ \hat{f}_3=\begin{bmatrix}0\\\hat{x}_3\\0\end{bmatrix},$$

$$\hat{f}_4=\begin{bmatrix}0\\\sin\hat{x}_1\\0\end{bmatrix},\ \hat{f}_5=\begin{bmatrix}0\\0\\-\hat{x}_3\end{bmatrix},\ \hat{f}_6=\begin{bmatrix}0\\0\\-\hat{x}_2\end{bmatrix}$$

$$g_1=\begin{bmatrix}0\\0\\1\end{bmatrix}$$

The estimated system states are given by the observer eqn (13), choosing $K_{obs}=100$ results in the following equations for $\hat{x}_2$, $\hat{x}_3$

$$\dot{\hat{x}}_2=-(\hat{\theta}_2^1\hat{x}_2)+\left(\frac{-30000}{\hat{\theta}_6^1}-\frac{300\hat{\theta}_5^1}{\hat{\theta}_6^1}\right)(x_1-\hat{x}_1)+\hat{\theta}_3^1\hat{x}_3+\hat{\theta}_4^1\sin(\hat{x}_1) \tag{47}$$

$$\dot{\hat{x}}_3=\hat{\theta}_1^2u-\hat{\theta}_6^1\hat{x}_2+300(x_1-\hat{x}_1)-\hat{\theta}_5^1\hat{x}_3 \tag{48}$$

The optimal control output u_{opt} is then given by:

$$\begin{aligned}u_{opt}=&-0.6842(x_1-y_{ref})+10.0115x_2+0.9933x_3\\&+8.7554(x_1-y_{ref})\cos(x_1-y_{ref})\\&-2.32(x_2-\hat{x}_2)\cos(x_1-y_{ref})\\&-1.6(x_3-\hat{x}_3)\cos(x_1-y_{ref})+0.001\sin(x_1-y_{ref})\end{aligned} \tag{49}$$

Simulation results

The simulation assumed nominal values for the physical parameters (*Table 1*). These values correspond to system parameters

$\theta_1^1, \theta_2^1, \theta_3^1, \theta_4^1,$

$\theta_5^1, \theta_6^1, \theta_1^2$

as shown in *Table 2*. The initial conditions used are shown in *Table 3*. *Figure 1* shows the response of the reference signal (solid line) and the output signal (dashed line) in case of applying typical feedback linearization. It is clear from the figure that although the output shows a very rapid and accurate tracking for the reference signal despite the different initial conditions for the signals (reference and output) and the mismatch in system parameters (nominal and estimated), it suffers from a high overshoot. The proposed optimal feedback linearization controller does not suffer from this drawback and to clarify this fact the corresponding response was plotted for this controller in *Figure 2*. The main purpose of applying the proposed optimal feedback linearization controller is the fact that the required control effort to achieve the same response (in this case better) is considerably less than the case of applying typical feedback linearization. This fact is illustrated in *Figure 3*.

The abovementioned simulations were conducted using Matlab [14].

Conclusion

The increasing requirement on robot tracking performance prompts the study of including motor dynamics in controller design, an approach made possible by today's ever advancing computer technology. The currently available robot controllers in this class require acceleration feedback and exact knowledge of both robot and motor dynamics. In this paper we presented a new approach addressing this problem by adopting adaptive feedback linearization techniques [12] and enhancing those techniques [12] to be applied in an optimal sense and without the need for an explicit update parameter based on the certainty equivalence principle. We achieved the first objective by exploiting the Hamilton–Jacobi equation and choosing an appropriate cost function; and the second was achieved by designing an observer that guarantees asymptotic stability for the estimation errors.

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