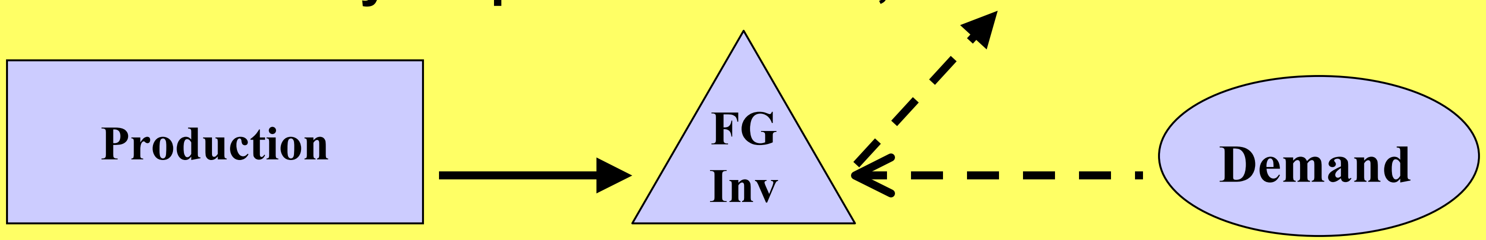
Base Stock Policy

If set up cost $S = 0$, then intuitively we set $Q = 1$

Let $S = R + 1 =$ base stock level

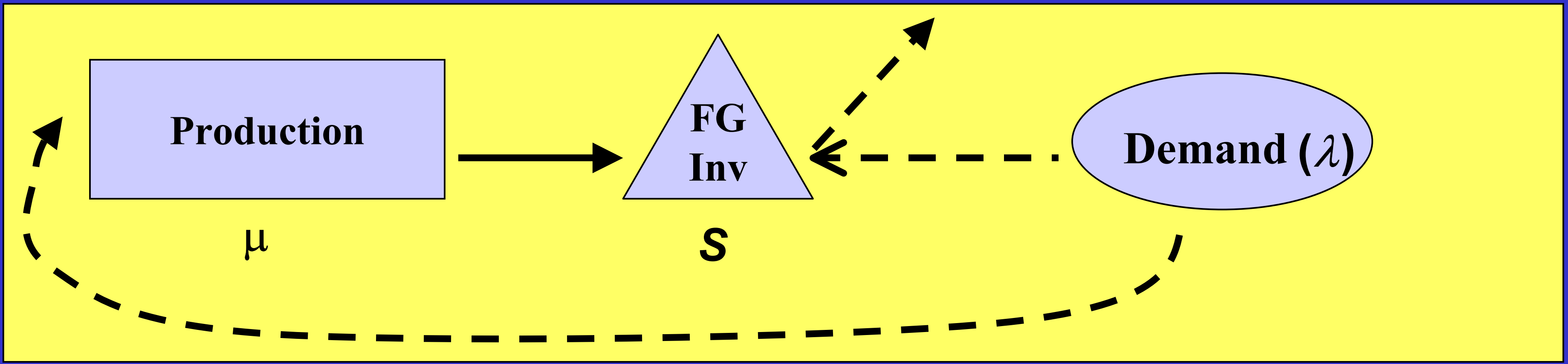
When inventory drops to $S - 1$ units, order 1 unit



- If manufacturer supplies many retailers, then lead times should be independent of retailer's ordering policy
- This is assumed in inventory theory
- Production facility is modeled as a $\bullet/M/\infty$ queue

- But if manufacturer supplies only one retailer, then lead times depend on the retailer's ordering policy
- This is assumed in queueing theory
- Production facility is modeled as a $\bullet/M/1$ queue
- Waiting time W in queueing system = lead time in inventory system

Make-to-stock Queue



$I(t) =$ *FG* inventory level
$Q(t) =$ items in production queue

$I(0) = S$
$Q(0) = 0$

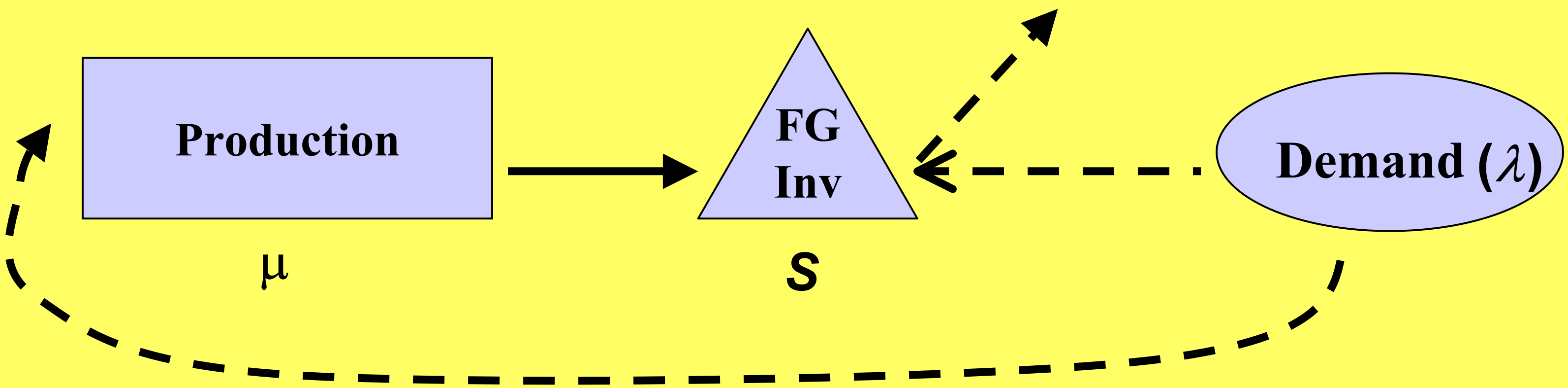
$I(t) + Q(t) = S$ for all $t \geq 0$

Poisson demand and exponential production times

$Q(t) =$ # in M/M/1 queue

$\Pr(Q(\infty) = n) = \rho^n (1 - \rho)$ geometric

Make-to-stock Queue Cont.



Instead, assume $Q(\infty) \sim$ exponential with mean $L = \dfrac{\rho}{1-\rho}$

Let b = backorder cost per unit inventory per unit time

h = holding cost per unit inventory per unit time

$$I(\infty) \sim f(x) = \begin{cases} L^{-1} e^{-L^{-1}(x-S)} & \text{if } x \le S \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Cost} = C(S) = -\int_{-\infty}^{0} bxL^{-1} e^{-L^{-1}(x-S)} dx + \int_{0}^{S} hxL^{-1} e^{-L^{-1}(x-S)} dx$$

$$\frac{dC(S)}{dS} = 0 \Rightarrow S^* = \frac{\rho}{1-\rho} \ell n\left(1 + \frac{b}{h}\right)$$

Periodic Review Inventory Systems

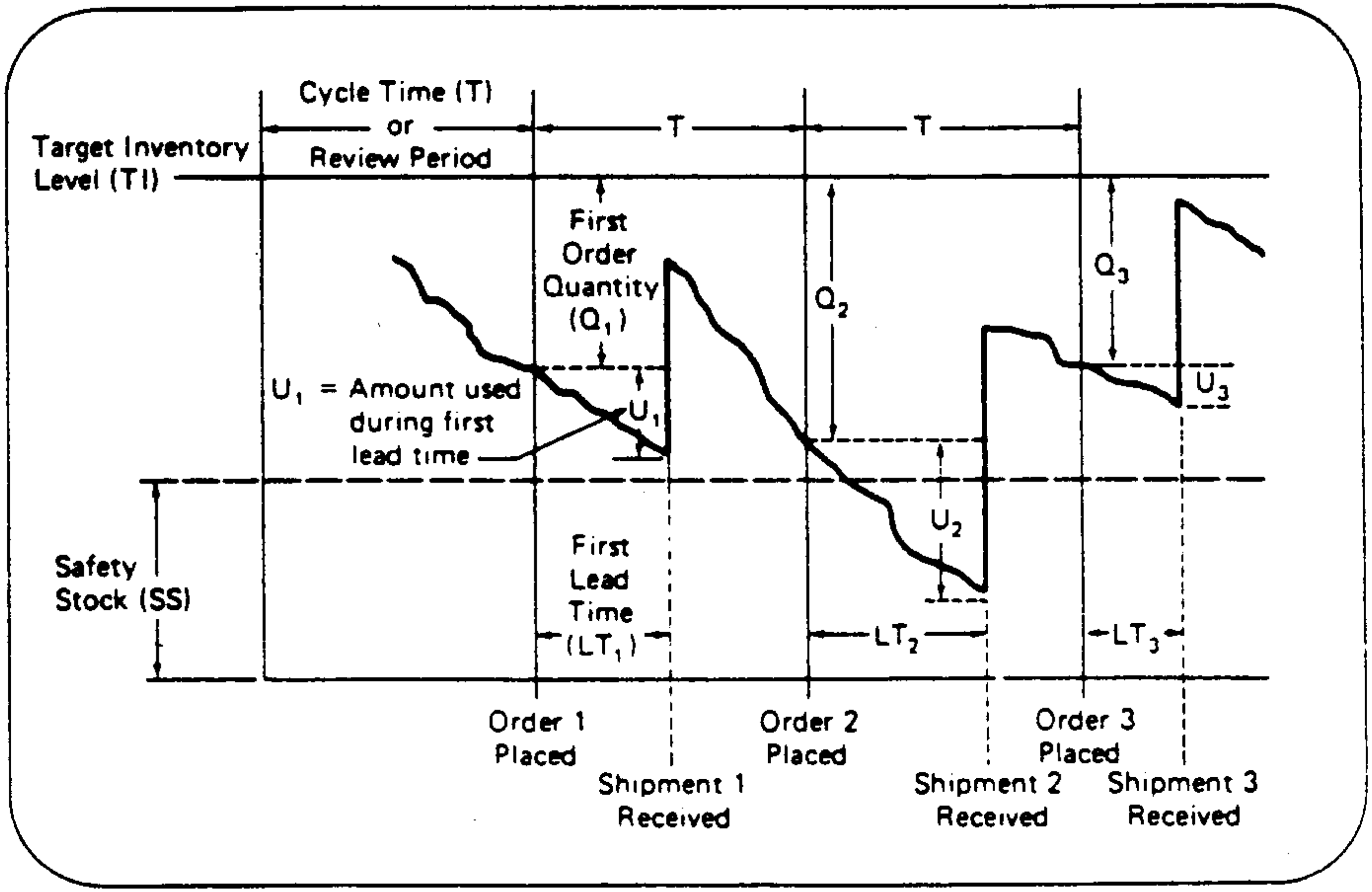
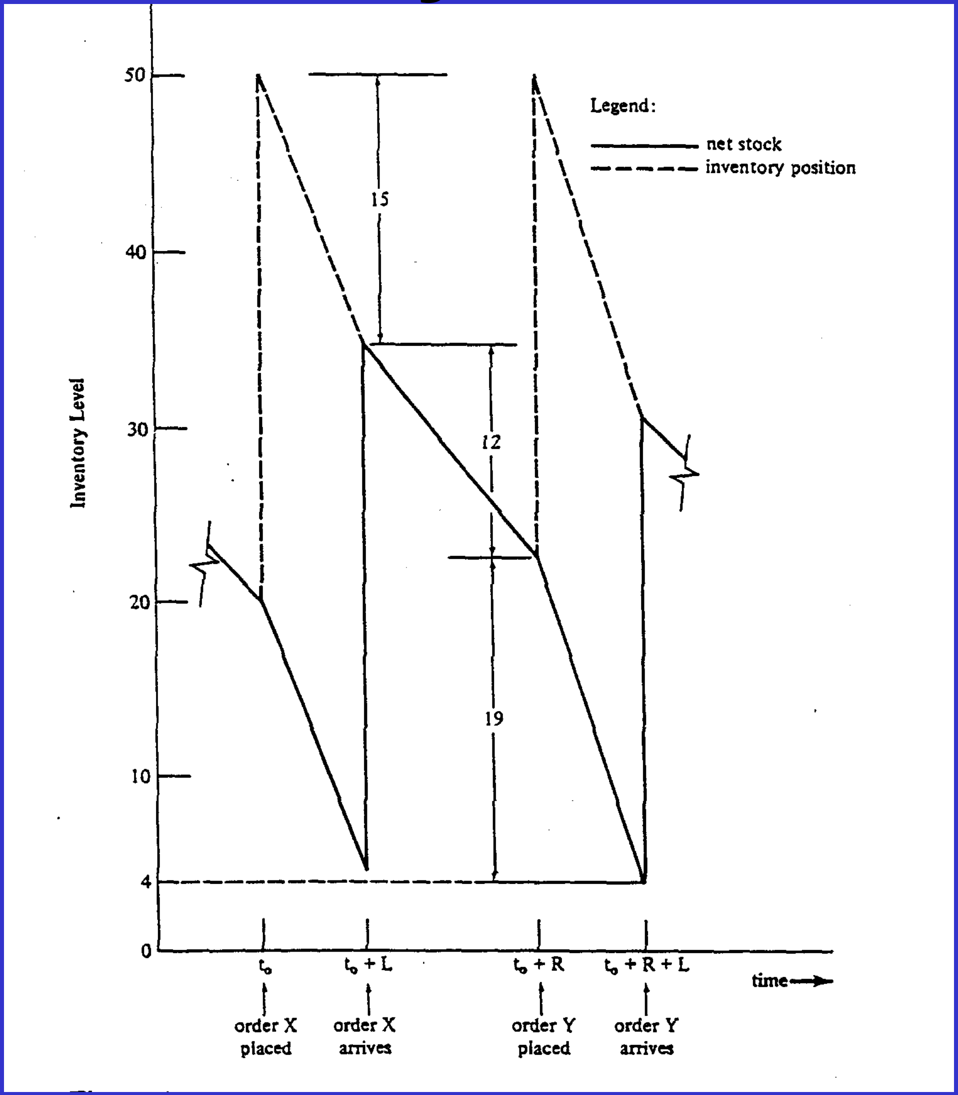


Figure 10-4 Level of inventory through time in a periodic review system: Order quantity = target inventory − inventory on hand.

Inventory Position



Key insight: the current order must satisfy demand until the next order arrives

(S,T) Policy

Every T time units, replenish your inventory position to S units

Let $T = \dfrac{EOQ}{\lambda}$

Let *DDLTRP* = demand durning lead time plus review period

- We need *S* units to satisfy *DDLTRP*
- Use newsboy model

Set *S* so that $\Pr(DDLTRP \leq S) = \dfrac{r - c}{r - s}$

Example

$L =$ deterministic lead time

$L_r = L + T = LT + RP$

$D_i = iid$ demand in each period with mean $E[D]$ and variance $\sigma^2[D]$

$$DDLTRP = D_1 + \ldots + D_{L_r}$$

$$E[DDLTRP] = L_r E[D]$$

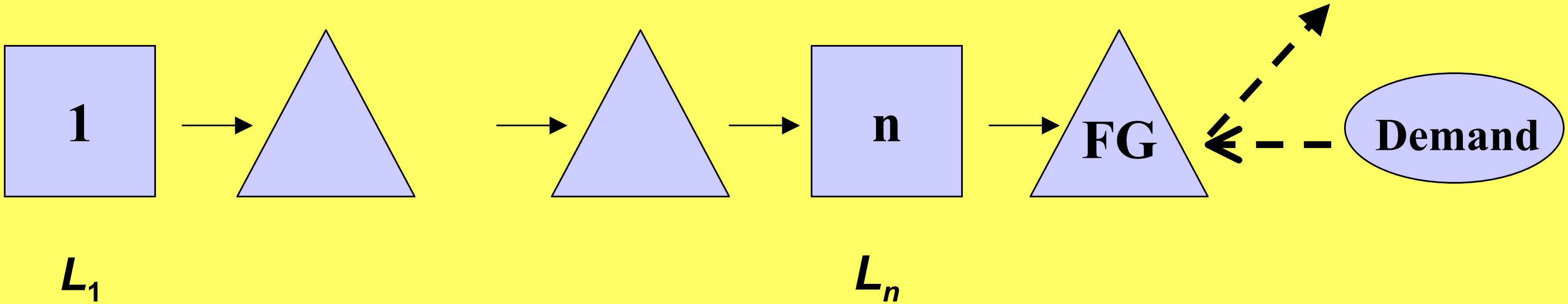
$$Var[DDLTRP] = L_r \sigma^2[D]$$

Set S so that $\Pr(DDLTRP) \le S$

$$= \Pr\left(\frac{DDLTRP - L_r E[D]}{\sqrt{L_r}\,\sigma[D]} \le \frac{S - L_r E[D]}{\sqrt{L_r}\,\sigma[D]}\right)$$

$$= \frac{r - c}{r - s}$$

Multi-echelon Inventory Systems



Question: How should each station replenish its inventory?

Answer: Track echelon inventory = total downstream inventory, but base stock levels are difficult to compute