The Laws of Forecasting

1. Forecasts are always wrong.
2. Forecasts always change.
3. The further into the future, the less reliable the forecast will be.

Forecasting is *statistics* . It is therefore powerful, but dangerous and easy to misuse.

Beware of black boxes!

Forecasting

Two Types of Forecasting

- **Explanatory**: Regression on causal factors
- **Time Series**: Regression on historical demand

D_t = actual demand in period t

F_t = forecast of demand in period t, given $D_1,\ldots,D_{t-1}$

Forecasting

- Use causal factors whenever they are available.
  - ⋆ Anticipated price changes, economic changes, competitor actions, etc.
- *Flexibility* is needed in anything that requires forecasts.
  - ⋆ Try to design the system to reduce lead times, so that shorter term forecasts are adequate.

Moving Average

$F_t =$ average of most recent n data points

$$= \frac{\sum_{i=t-n+1}^{t} d_i}{n}$$

High value of n: more stable, less responsive

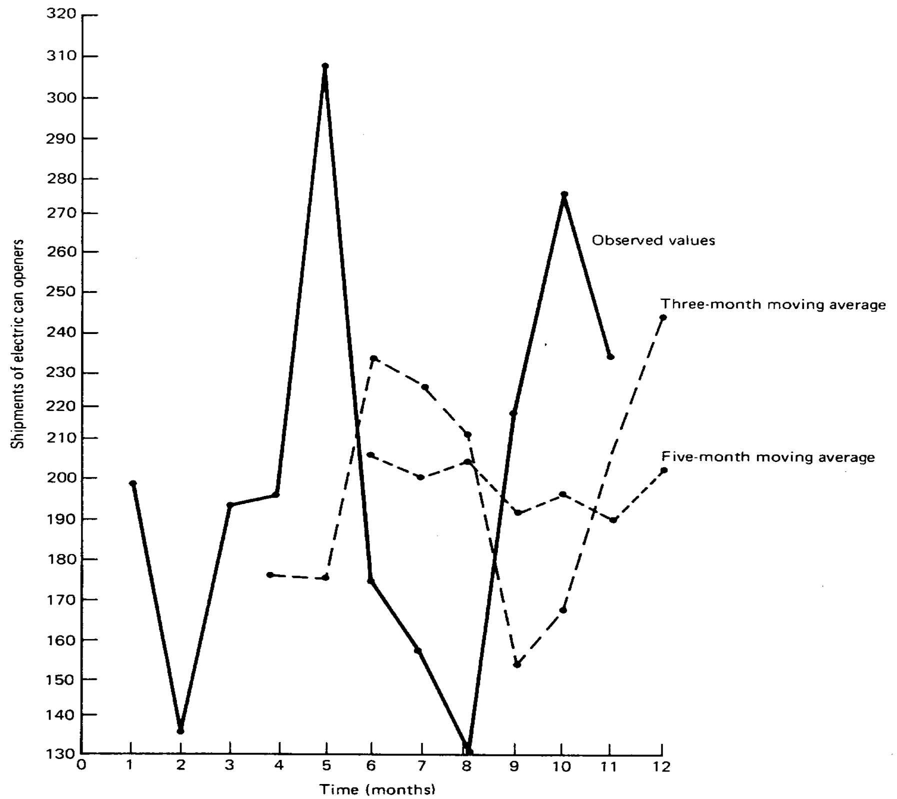


FIGURE 3-2 SHIPMENTS OF ELECTRIC CAN OPENERS—ACTUAL AND MOVING AVERAGE VALUES

Exponential Smoothing

$$F_t = \alpha D_{t-1} + (1-\alpha)F_{t-1}$$
$$= F_{t-1} + \alpha(D_{t-1} - F_{t-1})$$
$$= F_{t-1} + \alpha e_{t-1}$$

$$F_t = \alpha D_{t-1} + (1-\alpha)[\alpha D_{t-2} + (1-\alpha)F_{t-2}]$$
$$\vdots$$
$$= \alpha D_{t-1} + \alpha(1-\alpha)D_{t-2} + \alpha(1-\alpha)^2 D_{t-3} + \ldots$$

α **near 0** $\Rightarrow$ **stable, unresponsive**

α **near 1** $\Rightarrow$ **responsive to recent demand**

n (moving average) $\approx \dfrac{2-\alpha}{\alpha}$

$$\alpha \approx \frac{2}{n+1}$$

Exponential Smoothing

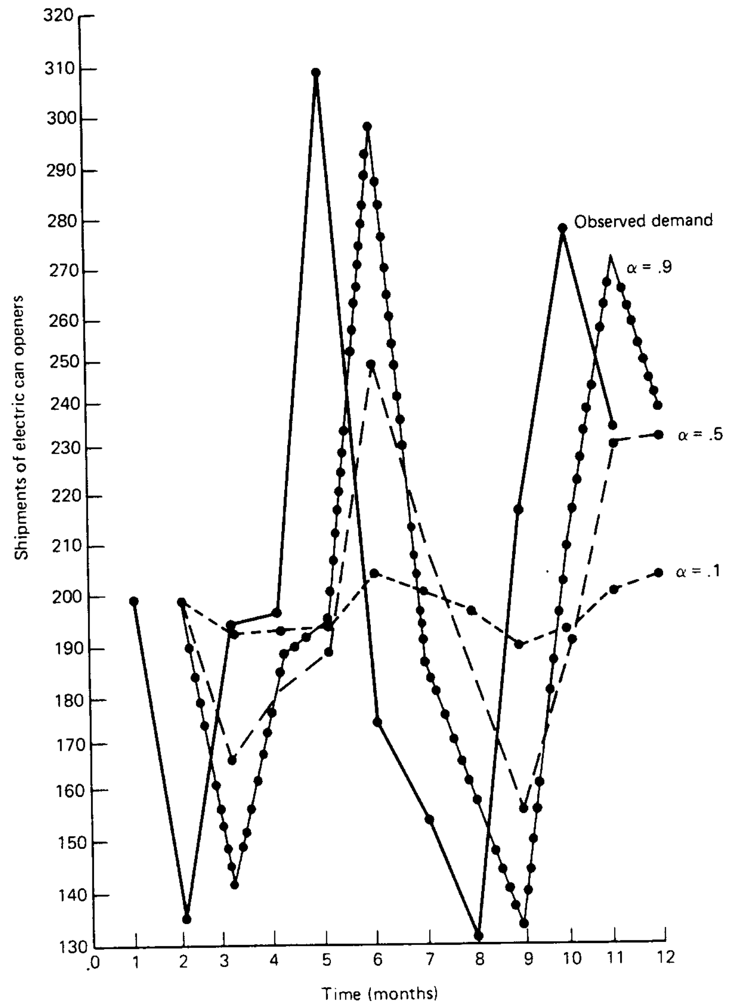


FIGURE 3-3 SHIPMENTS OF ELECTRIC CAN OPENERS—ACTUAL AND EXPONENTIAL SMOOTHING VALUES

Forecasting

- Lower values of α make the forecasts *more* stable and *less* responsive.

ARMA Process

Due to Box and Jenkins

$$F_{t+1} = \text{ linear function of } \underbrace{(D_1, \ldots, D_t)}_{\textbf{autoregressive}} \text{ and } \underbrace{(e_1, \ldots, e_t)}_{\textbf{moving average}}$$

See "Time Series Analysis : Forecasting and Control" By Box, Jenkins and Reinsel (1994)

Forecasting

The basic idea is to assume a model for the stochastic process X_t, and then to use observations on past X_t to determine the parameters of the model. Future values of X_t are then predicted by using the model.

- *Autoregressive:* It is assumed that X satisfies $X_t = \delta + \sum_{i=1}^{p} \phi_i X_{t-i} + A_t$ where δ, $\phi_1, \ldots, \phi_p$ are parameters and A_t is normal with zero mean.
- *Moving average:* It is assumed that X satisfies $X_t = \bar{X} + A_t - \sum_{i=1}^{q} \theta_i A_{t-i}$ where $\bar{X}$ is the mean of X, $\theta_1, \ldots, \theta_q$ are more parameters, and $A_0, A_1, \ldots, A_t$ are normal with zero mean on X_t.

Forecasting

ARMA combines both kinds of models:

- *ARMA:* $X_t = \delta + \sum_{i=1}^{p} \phi_i X_{t-i} + A_t$
  $- \sum_{i=1}^{q} \theta_i A_{t-i}$
- Parameters to determine: δ, $\phi_1, \ldots, \phi_p$ $\theta_1, \ldots, \theta_q$, p, q and the variance of A_i.

Forecasting

- The numerical analysis is difficult, especially when p and q must be determined from the data.
- The more parameters you have, the more data you need to estimate them.
- Many variations, including vector X.

From `http://www.itl.nist.gov/div898/handbook/index.htm`.

The Bass Model

m = fixed market size

$n(t)$ = instantaneous demand at time t

$N(t)$ = cumulative demand up to time t

p = coefficient of innovation

q = coefficient of imitation

$f(t)$ = rate of increase of fraction of buyers at time $t = \dfrac{n(t)}{m}$

$F(t)$ = cumulative fraction of buyers up to time $t = \dfrac{N(t)}{m}$

***Model*:**

hazard rate of fraction of purchasers

$$= \frac{f(t)}{1-F(t)} = p + q\frac{N(t)}{m}$$

$$n(t) = p(m - N(t)) + q\frac{N(t)}{m}(m - N(t))$$

Forecasting

- *Bass model:* Causal model of sales of a product with finite total demand under monopoly conditions.

$$n_t = \frac{dN_t}{dt} = (p + (q/m)N_t)(m - N_t)$$

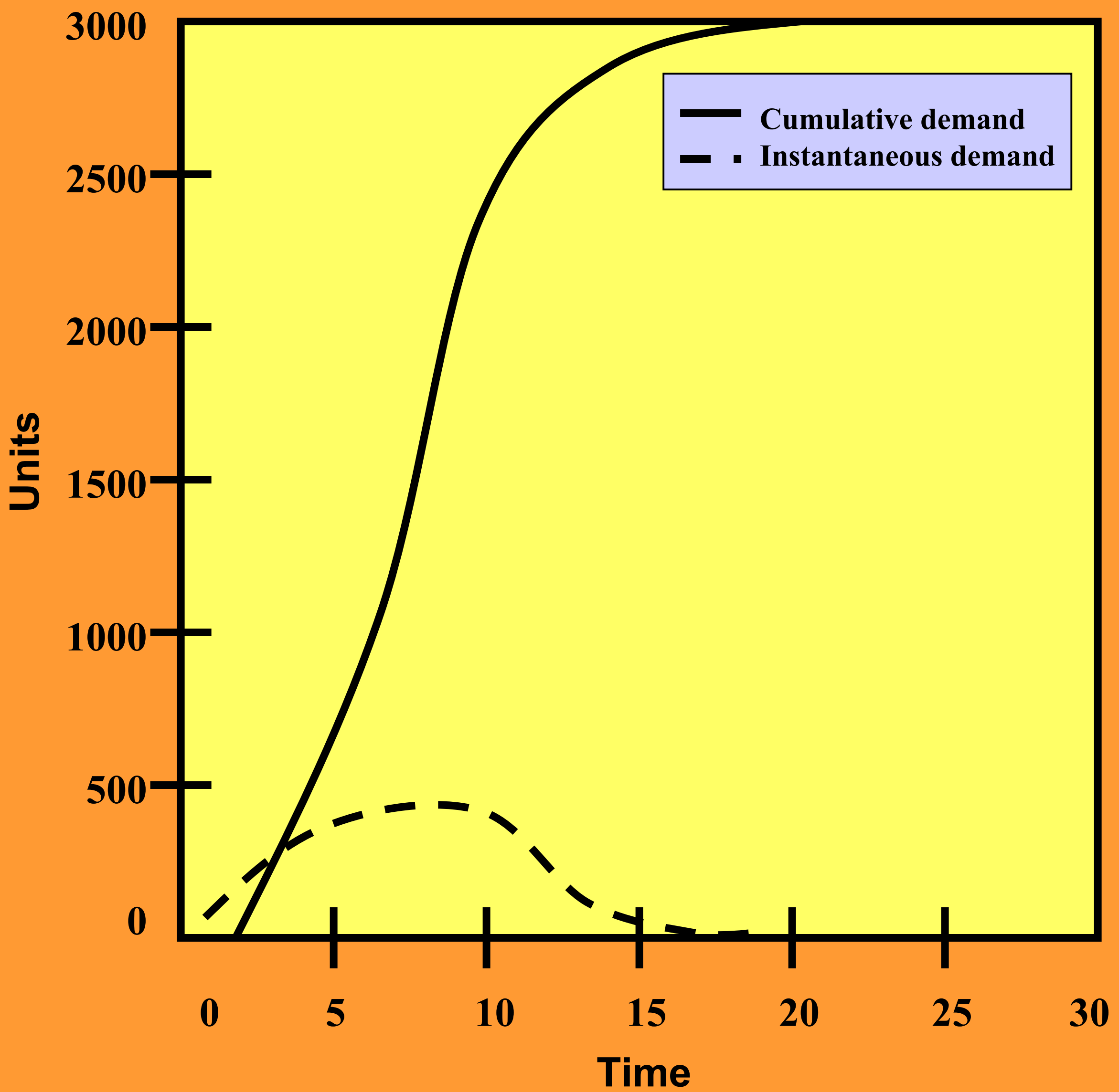
$$= p(m - N_t) + (qN_t/m)(m - N_t)$$

- Parameters are m, p, and q. $N_0 = 0$.
- $m - N_t$ is the portion of the total demand that has not yet been satisfied.

Solution to Bass Model

$$n(t) = \frac{mp(p+q)^2 e^{-(p+q)t}}{\left[p + qe^{-(p+q)t}\right]^2}$$

$$N(t) = \frac{m(1 - e^{-(p+q)t})}{1 + \frac{q}{p} e^{-(p+q)t}}$$



Cumulative and Instantaneous Demand in the Classical Bass Model

Comments on Bass Model

1) Good for short life cycle products

2) Can embed in DDLT heuristics,
but hard to get variance

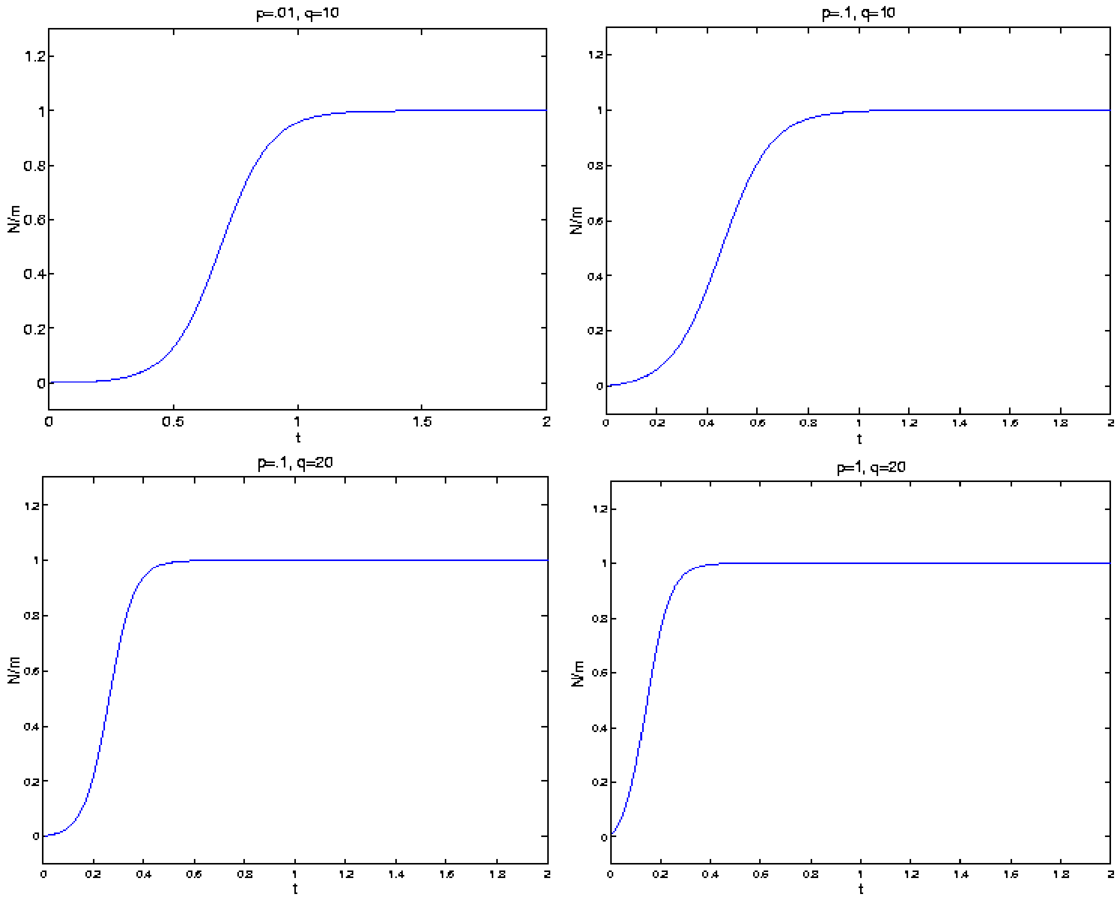
3) See Kurawarwala & Matsuo,
Operations Research 44, 131-150, 1996

Forecasting

- It is unusual to get an analytic solution to a nonlinear differential equation.
- *Procedure:* collect sales and other data and use it to estimate the parameters. Then predict N_t.
- Kurawarwala and Matsuo extended the model by including temporal variations in sales. Then they defined and solved an optimal production/inventory problem.

Forecasting

Bass model



The Martingale Model of Forecast Evolution

Notation:

D_t = actual demand in period t, $\{D_t\}$ stationary with $E(D_t) = \lambda$

$D_{t,t+i}$ = forecast of period $t+i$ demand made at start of period t

$\varepsilon_{t,t+i}$ = update in period $t+i$ forecast made at start of period t

$= D_{t,t+i} - D_{t-1,t+i}$

$\varepsilon_t = \left(\varepsilon_{t,t+i}\right)_{i=0}^{H}$ = forecast update vector at time t

The MMFE posits $\{\varepsilon_t\}$ forms an iid $N(0,\Sigma)$ sequence $\Sigma = [\sigma_{ij}]$

Note That:

- Σ can be estimated using historical forecast update data
- The MMFE is a descriptive model

Example:

Forecast Horizon = 2 months

$$\text{Var}\left(\varepsilon_{tt}, \varepsilon_{t,t+1}\right) = \Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}$$

If $\sigma_{12} = 0 \Rightarrow$ Demand is iid

$$\text{Var}(D_t) = \sigma_1^2 + \sigma_2^2 \quad \text{(update forecast twice)}$$

$$= \mathit{Var}(\text{February demand} \mid \text{January 1})$$

$$\sigma_1^2 = \mathit{Var}(\text{February demand} \mid \text{February1})$$

$$\frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2} = \text{fraction of variability predicted 1 month in advance}$$

$$\sigma_1^2 = 0 \Rightarrow \text{orders frozen 1 month in advance}$$

if $\sigma_{12} > 0 \Rightarrow$ new info. increases or decreases forecasts for both months

e.g., surge in market

if $\sigma_{12} < 0 \Rightarrow$ new info increases forecast for one month

new info decreases forcast for other month

e.g., quarterly demand known; timing of monthly demand is uncertain

Final Comments on MMFE

1) It is a model of an existing forecasting process, not a forecasting tool

2) It subsumes ARMA (stationary times series)

3) It can incorporate “black box” forecasting e.g., past demand, expert’s judgement, competitors’ actions, boss’ whims, etc.

4) It allows you to quantify different forecasts by estimating Σ from historical data

5) Optimal base stock levels

- inventory model
  (Graves et al., *Operations Research* 46,S35-S49, 1998)
- make-to-stock queue
  (Toktay and Wein, *Management Science*, 2001